Monotonic Switching Iterative Learning Control Method for a Class of Discrete Time Switched System

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Abstract: This paper deal with the problem of monotonic tracking convergence error (MC) of discrete time switched system. We begin our review by proposing the considered switched systems are operated during a finite time interval repetitively, and then the iterative learning control (ILC) scheme can be introduced between subsystems. Namely, the tracking error converges with non-zero constant initial error. After the switched system is transformed into a 2D switched system, sufficient conditions in terms of linear matrix inequalities (LMIs) are derived by using the l_2 -norm. It is shown that the tracking error converges monotonically to zero in the sense of l_2 -norm. A numerical simulation example is established shown the effectiveness of the proposed method.

Keywords: switched systems; iterative learning control (ILC); uadratic function; l_2 -norm; tracking control; linear matrix inequality

Introduction

The stability of hybrid systems is one of the most interesting and challenging topic in the modern engineering literature. A switched system is a hybrid dynamical system, which consists of a family of continuous-time or discrete-time subsystems and a rule that orchestrates the switching between them [1-3]. However, the stabilization problem of discrete switched systems with classic control law has been studied in [4-5-6 -7]. Specifying the asymptotic stability for switching systems with repetitive manner is an especially attracting problem in this work. Compared with the different results for synthesis issue by applying the classic control laws relatively few efforts are made for designing a controller to achieve the tracking error for switched systems [8-9-10]. Especially, for switched systems with repeated manner [11], ILC offers a systematic design that can improve the tracking performance by imposing the iterations in a varying time interval. Namely, the repetitive switching law is supervised by considering the learning gain at each subsystem. However, this idea ensuring the convergence of error between subsystem with non zero initial condition error.

The obtained formulation by applying ILC control is transformed into a synthesis problem of a special 2D switched system [12][13]. To achieve this desirable objective, the learning function that establishes the relationship between the tracking errors of two sequential iterations should be less than one [14], in the sense of some induced operator norms, where the bounded real lemma has been used for convergence analysis [15][16][20]. As a consequence, the tracking error can be guaranteed to converge monotonically in the sense of the l_2-norm. Then, the convergence conditions are expressed by LMIs which can determine the switching learning gains. Of particular interest in this paper, the switching learning can also eliminate the influence of the switching signal between subsystems that repeat each time. Also it clearly going to be required in at least some applications that the switching learning control laws is designed for switching discrete linear 2D repetitive processes which guarantee the stability between subsystems and also have the maximum possible convergence of error.

This paper is organized as follows: in section 2, based on the idea of expressing the dynamic of switched systems with repetitive switching manner. In section 3, a sufficient condition which guarantees the monotonic stability of the 2D switched system using S-ILC controller in the sense of the l2 norm. In section 4, the synthesis stability is derived in terms of a set of matrix inequalities. In section 5, Numerical examples are presented to illustrate the feasibility and the effectiveness of the proposed design algorithms in this paper. Finally, the conclusion of this paper is given in Section 6.

Notation. Notation used in the paper is standard. In general capital letters denote matrices. For two symmetric matrices, A and B $A>B$ means that $A-B$ is positive definite. A^T Denotes the transpose of A, $diag\{x,y,..\}$ denotes the diagonal matrix obtained from vectors or matrices $x,y,..$ When no confusion is possible, identity and null matrices will be denoted respectively by I and 0. Furthermore, in the case of partitioned symmetric matrices, the symbol $*$ denotes generically each of its symmetric blocks.

Problem Statement and Preliminaries

$$\begin{cases} x(t+1)=A_{\alpha(t)}x(t)+Bu(t) \\ \quad y(t)=Cx(t) \end{cases} x(0)=0,t\geq 0 \qquad (1)$$

where $x(t)\in R^n$ the state in R^n, $u(t)\in R^m$ is the control vector in R^m, $y(t)\in R^m$ is the output vector. $\alpha(t)$ is a switching rule defined by $\alpha(t):N\rightarrow I$ with $I = \{1, 2, .., l\}$.This means that the matrices (A_α,B,C) are allowed to take values, at an arbitrary discrete time, in the finite set;

$$\{(A_1,B,C),...,(A_l,B,C)\} \qquad (2)$$

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Note that the switching rule $\alpha(t)$ is operated during a repetitive manner in finite time intervals showing as follow:

$$\alpha(t)=k:\begin{cases} 1,t\in[0,t_1] \\ 2,t\in[t_1+1,t_2] \\ . \\ l,t\in[t_{l-1},T] \end{cases} \qquad (3)$$

k is defined as the switching operation for subsystem in the k^{th} iteration. In the sequel, we note:

$$A_k=A_{\alpha(t):=\{1,..l\}} \qquad (4)$$

Thus, (1) is expressed with the repetitive manner:

$$\begin{cases} x_k(t+1)=A_kx_k(t)+Bu_k(t) \\ \quad y_k(t)=Cx_k(t) \end{cases} x(0)=0,t\geq 0 \qquad (5)$$

where $x_k(t)\in R^n$ and $u_k(t)\in R^m$ are respectively the state and the control vector of the repetitive switched system, $t=0,....,T$, $k\in N$.

Remark 1. The proposed designing switching rules can be viewed as an extension form developed for 2D switched systems, by considering the arbitrary switching instants repeated for each iteration.

The following reasonable assumptions on the system (5) are imposed.

Assumption 1. The desired trajectory $y_d(t)$ is iteration invariant.

Assumption 2. Every operation begins at a varying initial condition at (iteration / subsystem), i.e

$$x_k(0)\neq x_k(t_1+1)\neq ...\neq x_k(T) \qquad (6)$$

In this paper, the control target is to find a control input sequence that guarantee the convergence of the output system profiles $y_k(t)$ to $y_d(t)$ as $k\rightarrow\infty$.

Main Results

2D Convergence analysis for linear switched system using ILC control

Consider the following updating ILC law:

$$u_{k+1}(t)=\beta u_k(t)+K_1^p\eta_{k+1}(t+1)+K_2^pe_k(t+1) \qquad (7)$$

and

$$\eta_{k+1}(t+1) = x_{k+1}(t) - \beta x_k(t) \tag{8}$$

where $e_k(t) = y_d(t) - y_k(t)$ is the output tracking error, $\eta_{k+1}(t+1)$ denotes the state vector computed to the cycle direction, K_1^p, K_2^p are switching learning gains with appropriately dimensioned matrices to be designed, with p is the index of learning gains for each subsystem, β positive scalar to be optimized.

Remark 2. The new idea of this paper that each subsystem is locally controlled by the switching learning gain and the ILC control is investigated when the initial conditions (6) are variable depending on the dynamics of the switched systems. It is obvious that the learning dynamics are more complex when the initial iterative state is different for each iteration (k^{th} switching). The local learning gains for each mode (denoted by p) ensure the locally stability between subsystem and limit the divergence of the output signals affecting the global stability of system (1).

Then clearly, (5) and (7) can be written as:

$$\begin{aligned}\eta_{k+1}(t+1) &= x_{k+1}(t) - \beta x_k(t) \\ &= \left(A_{k+1} + BK_1^p\right)\eta_{k+1}(t) \\ &\quad + \beta\left(A_{k+1} - A_k\right)x_{k+1}(t-1) \\ &\quad + BK_2^p e_k(t)\end{aligned} \tag{9}$$

$$x_{k+1}(t-1) = \eta_{k+1}(t) + \beta x_k(t-1) \tag{10}$$

$$\begin{aligned}e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\ &= -C\left(A_{k+1} + BK_1^p\right)\eta_{k+1}(t) \\ &\quad - C\beta\left(A_{k+1} - A_k\right)x_{k+1}(t-1) \\ &\quad + \left(\beta - CBK_2^p\right)e_k(t)\end{aligned} \tag{11}$$

the closed-loop system is given by the following form:

$$\begin{cases}\begin{bmatrix}\eta_{k+1}(t+1) \\ x_{k+1}(t-1)\end{bmatrix} = \begin{bmatrix}\left(A_{k+1} + BK_1^p\right) & \beta\left(A_{k+1} - A_k\right) \\ I & \beta\end{bmatrix} \\ \times\begin{bmatrix}\eta_{k+1}(t+1) \\ x_k(t-1)\end{bmatrix} + \begin{bmatrix}BK_2^p \\ 0\end{bmatrix} e_k(t) \\ \\ e_{k+1}(t) = \left[-C\left(A_{k+1} + BK_1^p\right) \quad -C\beta\left(A_{k+1} - A_k\right)\right] \\ \times\begin{bmatrix}\eta_{k+1}(t+1) \\ x_k(t-1)\end{bmatrix} + \left(\beta - CBK_2^p\right)e_k(t)\end{cases} \tag{12}$$

Let G_{bf} is the transfer from $e_{k+1}(t)$, to

$X_{k+1}(t) = \begin{bmatrix}\eta_{k+1}(t+1) \\ x_{k+1}(t-1)\end{bmatrix}$ such as:

$$\begin{aligned}G_{bf} &= \begin{bmatrix}A_{bf} & B_{bf} \\ C_{bf} & D_{bf}\end{bmatrix} \\ &= \begin{bmatrix}\begin{bmatrix}\left(A_{k+1} + BK_1^p\right) & \beta\left(A_{k+1} - A_k\right) \\ I & \beta\end{bmatrix} & \begin{bmatrix}BK_2^p \\ 0\end{bmatrix} \\ \left[-C\left(A_{k+1} + BK_1^p\right) \quad -C\beta\left(A_{k+1} - A_k\right)\right] & \left(\beta - CBK_2^p\right)\end{bmatrix}\end{aligned} \tag{13}$$

Or equivalent to $e_{k+1} = e_k G_{bf}$ where G_{bf} be a stable transfer function matrix of the closed loop system (12), with the switching learning gains K_1^p, K_2^p. The design objective for finding K_1^p, K_2^p, guarantying the monotonic convergence of error between iteration/subsystems, and to minimize the l_2 -norm of the closed-loop transfer function G_{bf} for e_k to e_{k+1} i.e. Satisfies:

$$\left\| G_{e_{k+1}/e_k} \right\|_\infty < \gamma \tag{14}$$

Then form (12), given this 2-D equivalent switched systems we have:

$$G_{bf} := \begin{cases}X_{k+1}(t+1) = A_{bf}X_{k+1}(t) + B_{bf}e_k(t) \\ e_{k+1}(t) = C_{bf}X_{k+1}(t) + D_{bf}e_k(t)\end{cases} \tag{15}$$

Remark 3. The closed switched ILC system (15) is essentially a 2D switched system form defined by the pass output and state vectors $e_{k+1}(t)$ and $X_{k+1}(t)$ respectively, since it includes two independent dynamic processes: one along the time axis t and the other along the iteration axis k. Therefore, the hybrid ILC switched system could be presented as a standard 2D Roesser system. In the following, how to achieve the 2D ILC switched dynamic formulation of the considered system based on Roesser systems is explained.

Tracking error convergence of the 2D switched system

In the sequel given the system (15) and respecting Assumption.1 and Assumption.2 , found the appropriate switching learning gain K_1^p, K_2^p such that the monotonic convergence in $e_k(t)$ is achieved, and the output error $e_{k+1}(t)$ converge to zero as $k \to \infty$, for $t = 0, \dots T$, $k \in N$.

Definition 1. [18] Given the system (5) and ILC controller (6), with "Assumption.1 and Assumption.2'' then, (14) is monotonically convergent in $e_k(t)$ if there exists $0<\gamma<1, \forall k \in N$, $y_d(t) \in R^m$ such that,

$$\left\| e_{k+1}(t) \right\|_2 < \gamma \left\| e_k(t) \right\| \tag{16}$$

where $e_k(t)$ the output error of system. In (16), the norm $\left\| e_k(t) \right\|_2$ is defined by

$$\left\| e_k(t) \right\|_2 = \sqrt{\left(\sum_{t=1}^{T} e_k^{\ T}(t) e_k(t) \right)} \tag{17}$$

Definition 2. [19] Discrete linear 2D switched system described by (14) is monotonic convergence between (iteration / subsystems), if there exists a block-diagonal matrix $P = diag(P_1, P_2) > 0$ such that the following LMI holds:

$$\Delta V_{k+1}(t) = V_{k+1}(t+1) - V_{k+1}(t) < 0 \tag{18}$$

Where,

$$V_{k+1}(t) = X_{k+1}^{\ T}(t) P X_{k+1}(t) \tag{19}$$

We will also require the following definitions. In order to ensure the l_2 norm holds it is required that the associated Hamiltonian defined by:

$$G_{bf}(k+1,t) = \Delta V_{k+1}(t) + e_{k+1}^T e_{k+1} - \gamma^2 e_k^T e_k \tag{20}$$

Satisfies $G_{bf}(k+1,t) < 0$.

We present now a useful lemma used in the proofs later in the paper.

Lemma 1. [8] (Schur Complement). Assume W, L and V are given matrices with appropriate dimensions, where W, V are positive definite symmetric matrices. Then

$$L^T V L - W < 0 \tag{21}$$

if and only if

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \tag{22}$$

or

$$\begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0 \tag{23}$$

Monotonically Convergent condition of the 2D switched system

In this section we shall establish servall versions of 2D bounded real lemma which provides a link between the H_∞ noise attenuation property of 2D switched system (15) and the solution of certain reccarti equation or inequality. The 2D switched system is described in a Roesser model with boundary condition.

Theorem 1. For given scalar $0<\gamma<1$, the system (4) with ILC updating law (7), then (15) is Monotonically Convergent in $e_k(t)$, if there is symmetric positive matrix X_1, X_2, and the matrix N_1^p, K_2^p with appropriate dimensions, and a scalar $\beta > 0$, such that the following LMI conditions are satisfied:

$$\begin{bmatrix} -X_1 & 0 & 0 & \begin{matrix} X_1 A_{k+1}^T \\ +\left(N_1^p\right)^T B^T \end{matrix} & X_2 & \begin{matrix} -X_1 A_{k+1}^T C^T \\ -\left(N_1^p\right)^T B^T C^T \end{matrix} \\ * & -X_2 & 0 & \beta X_1 \begin{pmatrix} A_{k+1} \\ -A_k \end{pmatrix}^T & X_2 \beta & -\beta X_2 \begin{pmatrix} A_{k+1} \\ -A_k \end{pmatrix}^T C^T \\ * & * & -\gamma^2 I & \left(B K_2^p\right)^T & 0 & \begin{pmatrix} \beta \\ -C B K_2^p \end{pmatrix}^T \\ * & * & * & -X_1 & 0 & 0 \\ * & * & * & * & -X_2 & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{24}$$

then:

- *The output error* $e_k(t)$ *converges to zero for al* $t \in [0, T]$ *as* k *goes to infinity;*
- *The monotonic convergence of the output tracking error* l_2 *norm between subsystems is achieved along the (time/iteration) direction for all* k, t.
- *the gain matrices of the hybrid ILC can be obtained by solving LMI (24) as:*

$$K_1^p = N_1^p \left(X_1\right)^{-1} \tag{25}$$

Proof: First, we consider the increased 2D switched system (15), if $X_{k+1}(t+1)$ is the input signal and $e_{k+1}(t)$ is the output. Then, we introduce the following quadratic performance function for the given scalar $0<\gamma<1$.

$$J(\gamma,k)=\sum_{t=1}^{T}\left[e_{k+1}{}^{T}(t)e_{k+1}(t)-\gamma^2 e_k{}^{T}(t)e_k(t)\right]\quad(26)$$

From (13) we can conclude that the system (14) is Monotonically Convergent in $e_k(t)$ if,

$$J(\gamma,k)<0,\forall k=1,2,..\forall\gamma\in(0,1]\qquad(27)$$

Here, using the same Lyapunov functional candidate as in (16), we get:

$$\Delta V_{k+1}(t)=V_{k+1}(t+1)-V_{k+1}(t)$$
$$=\begin{bmatrix}X_{k+1}^T\\ e_{k+1}^T\end{bmatrix}^T\left(\begin{bmatrix}A_{11}^T & A_{21}^T\\ A_{12}^T & A_{22}^T\\ B_{11}^T & B_{12}^T\end{bmatrix}\begin{bmatrix}P_1 & 0\\ 0 & P_2\end{bmatrix}\begin{bmatrix}A_{11} & A_{12} & B_{11}\\ A_{21} & A_{22} & B_{12}\end{bmatrix}-\begin{bmatrix}P_1 & 0 & 0\\ 0 & P_2 & 0\\ 0 & 0 & 0\end{bmatrix}\right)\begin{bmatrix}X_{k+1}\\ e_{k+1}\end{bmatrix}<0$$

(28)

where ,

$$\begin{aligned}&A_{11}=\left(A_{k+1}+BK_1^p\right),A_{12}=\beta\left(A_{k+1}-A_k\right)\\&A_{21}=I,A_{22}=\beta\end{aligned}\qquad(29)$$

$$B_{11}=BK_2^p,B_{12}=0\qquad(30)$$

Define,

$$C_{11}=-C\left(A_{k+1}+BK_1^p\right),C_{12}=-C\left(A_{k+1}-A_k\right)\qquad(31)$$

$$D=\left(\beta-CBK_2^p\right)\qquad(32)$$

To establish the weighted l_2-norm, we choose the same quadratic function as in (26), we get:

$$e_{k+1}{}^{T}(t)e_{k+1}(t)-\gamma^2 e_k{}^{T}(t)e_k(t)$$
$$=\begin{bmatrix}X_{k+1}^T\\ e_{k+1}^T\end{bmatrix}\begin{bmatrix}C_{11}^TC_{11} & C_{11}^TC_{12} & C_{11}^TD\\ * & C_{12}^TC_{12} & C_{12}^TD\\ * & * & D^TD-\gamma^2 I\end{bmatrix}\begin{bmatrix}X_{k+1}\\ e_{k+1}\end{bmatrix}$$

(33)

With help of (28), (33), we can rewrite (20) as:

$$\begin{bmatrix}X_{k+1}^T\\ e_{k+1}^T\end{bmatrix}^T\left(\begin{bmatrix}A_{11}^T & A_{21}^T & C_{11}^T\\ A_{12}^T & A_{22}^T & C_{12}^T\\ B_{11}^T & B_{12}^T & D^T\end{bmatrix}\begin{bmatrix}P_1 & 0 & 0\\ 0 & P_2 & 0\\ 0 & 0 & I\end{bmatrix}\times\begin{bmatrix}A_{11} & A_{12} & B_{11}\\ A_{21} & A_{22} & B_{12}\\ C_{11} & C_{12} & D\end{bmatrix}-\begin{bmatrix}P_1 & 0 & 0\\ 0 & P_2 & 0\\ 0 & 0 & \gamma^2 I\end{bmatrix}\right)\begin{bmatrix}X_{k+1}\\ e_{k+1}\end{bmatrix}<0\quad(34)$$

Following the proof line of Lemma 1, we can get:

$$\begin{bmatrix}-P_1 & 0 & 0 & A_{11}^T & A_{21}^T & C_{11}^T\\ * & -P_2 & 0 & A_{12}^T & A_{22}^T & C_{12}^T\\ * & * & -\gamma^2 I & B_{11}^T & B_{12}^T & D^T\\ * & * & * & -P_1 & 0 & 0\\ * & * & * & * & -P_2 & 0\\ * & * & * & * & * & -I\end{bmatrix}<0\qquad(35)$$

Given $P=X^{-1}$, where $X=diag(X_1,X_2)$ .Pre- and post-multiplying (35) $diag(X,I,I,I,I)$, we obtain (24).

Remark 4. By analogy with the 2D discrete time repetitive process [18], the first major result here is Theorem1. below which gives and H_∞ condition for the stability between subsystems in case to 2D switched system (15). Also it is clearly going to be required in at least some applications that control laws are designed for discrete linear repetitive processes which guarantee stability between subsystems and minimized the tracking error of 2D switched system.

Numerical Example

Success method

To prove the efficiency of the proposed conditions (20), a numerical evaluation is given in this section. The result obtained using the Theorem 1 is compared to the methods developed in [16] and summarize in the Table 1. For fixed values of (N; n; m; r), we generate randomly 20 switched systems of the form (1).

For each switched system, we try to compute a stabilizing switched learning control using two methods. By using the Matlab LMI Control Toolbox to check the feasibility of the LMI conditions, we introduce a counter (Method 1(theorem 1), Success Method 2 [16]) which is increased if the corresponding method succeeds in providing an switching ILC One can see that our proposed S-ILC synthesis conditions given in Theorem 1 reduce significantly the conservatism compared to [16].

Table 1. Feasibility Method.

Switched system	***Success***	$l=2$
$n=2, m=1, r=1$	Method 1 Method 2	17 10
$n=3, m=2, r=2$	Method 1 Method 2	14 6
$n=4, m=2, r=2$	Method 1 Method 2	13 2

Monotonic tracking error

In this section, an example is used to verify our conclusions. Let us consider the linear discrete-time switched System (1), which contains two subsystems:

$$A_1=\begin{bmatrix}0 & 1\\ 0.125 & -0.2\end{bmatrix}, A_2=\begin{bmatrix}-0.25 & 1\\ 0 & -0.3\end{bmatrix}$$

$$B=\begin{bmatrix}0\\ 1\end{bmatrix}, C=\begin{bmatrix}0.25 & 1\end{bmatrix}$$

and the desired trajectory,

$$y_d(t)=\sin(3t)+0.5\sin(2t)+0.5\sin(3t), t\in[0,40]$$

The initial conditions $x_k(0)=0$, and the control law $u_k(t)$ are given in (7). In this case, we assume the repetitive switching sequence is operated as follow:

$$\alpha(t)=k:\begin{cases}1, t\in[0,2]\\ 2, t\in[3,5]\\ .\\ l, t\in[38,40]\end{cases}$$

The aim is to determine the maximum limits of the monotonic convergence error in the plane $(\beta,\ \gamma)$ from (method 1 (theorem 1) and method 2 ([16]). For illustration purposes, the resulting gains obtained by applying (method 1 and method 2) are listed in table 2.

The results described in table 2, shown the efficiency of an S-ILC given in LMI (24) compared to condition [16], and guarantee the upper bound γ,β. Figure 1 shows that $e_{k+1}(t)$ is convergent to 0 as iteration number increased.

Figure 2 and Figure 3 show the output and input variables at the different iteration number. Obviously, system output profiles and control input profiles are also convergent on iteration domain.

Table 2. Numerical results.

Approach	***Controller gains***	*γ*	*β*
Method1	$K_1^1=[0.1458\ \ -0.98]$ $K_1^2=[-0.9944\ \ -0.98]$ $K_2^1=0.063$ $K_2^1=0.004$	0.96	0.77
Method2	$K_1^1=[0.286\ \ -1.265]$ $K_1^2=[-0.999\ \ -1.14]$ $K_2^1=0.2$ $K_2^1=0.005$	0.46	0.53

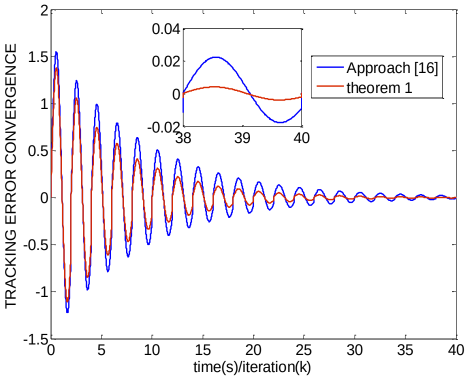

Figure 1. Decreasing of the output tracking error between subsystems (theorem1, at 19 iteration; approach [16] at 24 iteration).

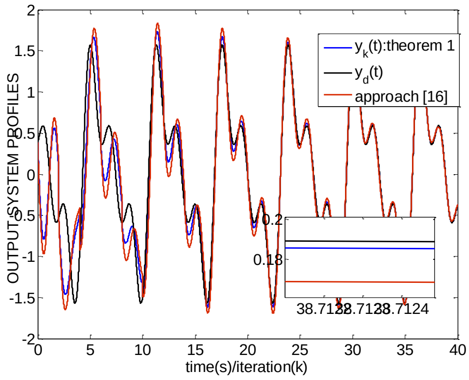

Figure 2. The time and iteration evolution of the desired trajectory $y_d(t)$ and the output $y_k(t)$.

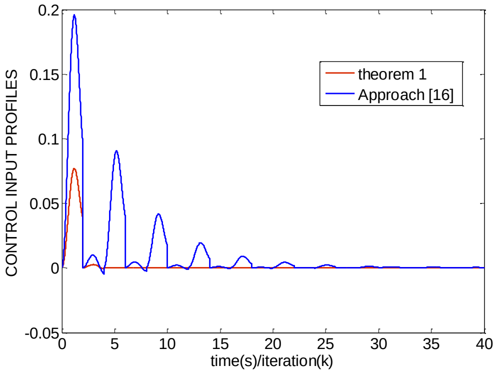

Figure 3. The control input evolution $u_k(t)$.

In the sequel, we tested the optimum value $\left(\gamma_{opt}, \beta_{opt}\right)$, computing by the proposed algorithm (LMI 20) that ensuring the minimum time of the error convergence listed in figure 4, 5.

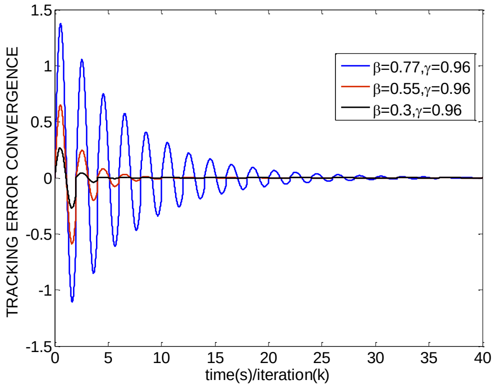

Figure 4. Decreasing of the output tracking error between subsystems as iteration number increased, for the marginal value of β .

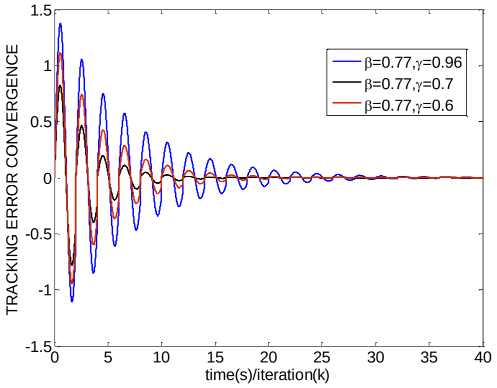

Figure 5. Decreasing of the output tracking error between subsystems as iteration number increased, for the marginal value of γ .

It is worth noting that the given conditions in Theorem 1 are obtained by using the lyapunov approach over the time switching and the relaxed parameters β play a key role to guarantee the monotonic speed convergence of system for minimum number of iteration after two (iteration/subsystems).

The previous results show the effectiveness of the S-ILC method compared to other conventional control. The stability analysis shows that the output error l_2 - norm is MC as the switching learning process proceeds from (trial/subsystem) to the next, that the previous input signal $x_k(t)$ is only bounded in amplitude. This approach ensuring a very important result to limit the divergence of the switching signal between subsystems and ensuring the global stability.

Conclusion

In this paper, we have presented the S-ILC tracking control design for switched systems with repetitive manner, based on the use of the l_2 - norm. We have shown that the control law design is updating with non zero initial condition, depending on the dynamic of switched system. Furthermore, we have explored this idea to minimize the switching signals between subsystem, and guarantee the asymptotic stability and guaranteed the convergence error along the (time-subsystem / iteration). Sufficient conditions for the existence of such controller are formulated in terms of a set of LMI. A numerical example is given to illustrate the effectiveness of the proposed methods.

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