



Fatigue Minimization Work Shift Scheduling for Air Traffic Controllers

Ta-Chung Wang^{1, *} and Guo-Chuan Ke²

¹*Institute of Civil Aviation, National Cheng-Kung University, Taiwan*

²*Department of Aeronautics and Astronautics, National Cheng-Kung University, Taiwan*

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*Corresponding author: tchung@mail.ncku.edu.tw

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Abstract: It is common for Air Traffic Controllers to control air traffic during the night and to experience fatigue. Although fatigue is not the direct cause of aviation accidents, 21 percent of accidents are fatigue-related. Therefore countries and companies have tried to regulate work hours to avoid extreme fatigue, thus decreasing human error resulting from fatigue. However, these regulations may not reflect that actual fatigue variation and fatigue levels can be decreased still more by scheduling appropriately. This paper focuses on optimal work shift scheduling to reduce air traffic controller fatigue. First, a mathematical model is established to describe fatigue levels. The objective function is to reduce the fatigue peak produced by work shifts as much as possible. Various constraints, such as holidays and manpower requirements are considered. The optimization problem is then solved using integer programming. We take a sample schedule and draw conclusions by comparing our results with the original fatigue levels.

Keywords: Scheduling; Integer programming; Air traffic controller; Fatigue model

Introduction

Air traffic control is the process by which aircraft are directed by air traffic controllers (ATCs) in order to separate air traffic in order to prevent collisions. ATCs need to provide information to pilots while aircraft are taxiing on the ground or flying in the air in order to keep the air traffic unimpeded and to enhance aviation safety. On the basis of the above requirements, air traffic control must be provided 24-hours a day in many airports, which leads to controllers working in shifts.

Fatigue-related problems cost America an estimated 18 billion a year in terms of lost productivity. Fatigue-related drowsiness on the highways contributes to more than 1,500 fatalities, 100,000 crashes, and 76,000 injuries annually [1]. As for aviation, 21 percent of all reported incidents (reported by both pilots and controllers) in the Aviation Safety Reporting System (ASRS) are fatigue-related [2]. These show the importance for maintaining a good working state in order to reduce accidents. It is commonly known that shift work such as that occurring in air traffic control is

associated with a high prevalence of fatigue [3]. Therefore, it is important to schedule an appropriate shift table that takes the reduction of fatigue into consideration.

Scheduling the shifts for air traffic controllers is a challenge because ATCs and their superiors may have conflicting opinions. Proper scheduling approaches have been debated since the 1970s, and up to now, there has still been no clear direction. Nevertheless, some guidelines have been suggested regarding scheduling.

Clockwise rotation schedule is a popular scheduling method. [4] evaluated shift-specific sleep, general activity levels, mood and cognitive performance of air traffic controllers who worked a forward 2-2-2 rapid rotation schedule. ATCs recorded their sleep, oral temperature and subjective fatigue levels, and subjects completed the Profile of Mood States questionnaire. Analyses were made on the basis of duty shift, post-shift, day of shift, and duty location and statistical significance was evaluated regarding scored sleep, reported sleep, measured fatigue and confusion on the above referenced shifts. The result showed that ATCs on the night-shift of a forward rapid rotation shift schedule appeared to be falling asleep and report increased confusion and fatigue.



Some research, like that of [3], has been concerned about different types of shift work. In [3], different work schedules (day work, three-shifts, and irregular shift work) were investigated to determine whether they are related to increasing fatigue over time. Data from nine consecutive four-monthly self-administered questionnaires from the Maastricht Cohort Study were used. The Maastricht Cohort Study is part of the Netherlands concerted research action on "Fatigue at work" granted by the Netherlands Organization for Scientific Research. According to the results, the prevalence of fatigue was 18.1% in day workers, 28.6% in three-shifts, 23.7% in five-shifts, and 19.1% in irregular shift workers.

Constraint programming is a powerful tool for finding feasible solutions to rostering problems. [5] used both soft constraints and hard constraints to describe nurse shift scheduling problem constraints. Hard and soft constraints are defined according to the importance of each constraint. The objective is to find high quality shift assignments that nurses approve of. That is, the objective function measures how much the solution deviates from the soft constraints. Two different techniques, the sequential technique and the equivalent weights technique, were proposed to solve the nurse scheduling problem. Using similar formulations, [6] proposed a satisfiability approach to solve nurse scheduling problems using comparisons with a simulated annealing approach and a genetic algorithm approach. [7] considered the problem of managing work shifts for nuclear power plants. The objective function was to minimize the work time differences among all employees, and a heuristic method was proposed to solve the problem. Human factors such as work and rest time regulations were considered in the constraints. [8] used the mixed-integer programming to solve a power system maintenance scheduling problem. The objective was to select the safest plan for a generator's outage schedule for the purpose of maintenance. The resulting schedule allowed reliable system operation. [8] developed a technique which considered an optimal linearized set of power flow equations. The results showed that representing transmission influence is essential for the

establishment of a sound set of scheduled outages for a system's generators. [9] later added uncertainty considerations into the power system scheduling problem. A fuzzy 0-1 integer programming model was adopted to find the minimum violation solution. [10] used a model similar to that of [9] and solved the problem using enumeration based 0-1 integer programming. All the above formulations focus on minimizing the violation of constraints or the work time difference of each person or machine. None of the existing methods tried to take fatigue variation into consideration. However, different shifts with the same amount of work time may have different workloads, and hence the resultant fatigue increments are different. This is especially true for ATC scheduling since the number of aircraft needed to be served varies depending on the time of day. Therefore, in this paper, a fatigue model is proposed to reflect the differences in different work shifts. The proposed fatigue model is such that it captures the fatigue variation characteristics during work and rest and is simple enough to be integrated into an optimization problem.

This paper focuses on scheduling appropriate shifts to reduce ATC peak fatigue levels in order to help ATCs work efficiently in good emotional states. The fatigue variations of ATCs are modeled by first order ordinary differential equations. Before scheduling, all ATCs are able to determine their own days off. Several constraints, such as work-hour restrictions, manpower requirements, and holidays are considered. Using this approach, we can directly minimize the fatigue of all workers while satisfying various regulatory and holiday constraints.

ATC shift scheduling problem

Fatigue modelling

To ensure that employees are at proper alertness levels when working in shifts, we need to be able to predict the fatigue level of each employee. Fatigue is generally accepted to be under the influence of prior sleep history, time spent at work and length of time spent awake [11]. The research conducted by [12] also showed that in addition to sleep length, wakefulness and work hours, workload significantly influences fatigue. Therefore, in our model, both the effects of workload and work hours are considered. The following are some mathematical models related to fatigue modeling.

The *Fatigue Audit InterDyne (FAID) system* [13] is a mathematical algorithm based on duration of work and rest periods. Its objective is to allow companies to assess and compare previous, current and possible future work schedules in terms of predicted work related fatigue. This

Ta-Chung Wang received the B. S. and M. S. degrees in mechanical engineering from National Taiwan University, Taipei, Taiwan, in 1995 and 2000, respectively. He received the Ph.D. degree in aeronautics and astronautics from Stanford University, Stanford, CA, in 2007. Currently, he is an assistant Professor of Institute of Civil Aviation, National Cheng-Kung University, Tainan, Taiwan. He teaches course in multivariate analysis, electronics and engineering mathematics. His research interests include operations research in the aviation industry and solving large problems using parallel approaches and nonlinear system analysis.

Guo-Chuan Ke received the B. S. and M. S. degrees in aeronautics and astronautics from National Cheng-Kung University, Tainan, Taipei.



is achieved by inputting work start and end times into the program. The output will be relative fatigue scores. The product is available from InterDynamics Pty Ltd, Adelaide, Australia.

The *US Army's Sleep Management System* [14] is another mathematical model based on work/rest periods and circadian cycles. A mathematical model to predict human performance on the basis of prior sleep was developed by US Army medical researchers at Walter Reed Army Institute of Research (WRAIR) [14, 15]. This model is implemented in a wrist-activity monitor called "Sleep Watch." The Sleep Watch system includes a wrist-worn piezo electric chip activity monitor and recorder that records the wearer's activities and the amount of sleep obtained over several days. This system inputs the measures with circadian periodicity variables into a sleep and performance predictive algorithm. At any point in time, the algorithm estimates how much in need of sleep the wearer is.

The *Three-Process Model of Alertness* [16] is another model for predicting alertness/performance. This model uses sleep data as input and outputs the predicted alertness value for each worker. Several exponential components relating to alertness and sleep inertia as well as a circadian rhythm alertness component are considered in this model.

Fatigue model for ATC scheduling

In order to manage fatigue, we need to know how much fatigue people experience. Therefore, a fatigue model to represent and quantify fatigue levels is required before constructing the optimization problem. Our fatigue model is inspired by the FAID system. The FAID model is constructed using a linear component (length of work period) and a sinusoidal component (circadian timing of work period) which is nonlinear and hence is not suitable for constructing constraints in the shift scheduling problem. The primary concept of FAID is to view a duty schedule as a time-varying function by which an individual is considered in one of two states: work or non-work. Each state can be considered as an input, from which a continuously varying fatigue/recovery result is the output.

FAID outputs fatigue scores in one of five ranges:

- **STANDARD** (0-40): The upper limit of this range is similar to the maximum fatigue score produced by the standard 9-5, Monday-Friday work week.
- **MODERATE** (40-80): The upper limit of this range is similar to the maximum fatigue score produced by a forward-rotating schedule (morning, afternoon, and night) with five consecutive 8-hour work periods followed by 2 days off.
- **HIGH** (80-100): The upper limit of this range is

similar to the maximum fatigue score produced by a forward-rotating schedule (morning, afternoon, and night) with six consecutive 8-hour work periods followed by 1 day off.

- **VERY HIGH** (100-120): The upper limit of this range is similar to the maximum fatigue score produced by a schedule that rotates through two 12-hour day shifts, 2 days off, two 12-hour night shifts, and 2 days off.
- **EXTREME** (>120): Fatigue scores of this magnitude are similar to those produced by a permanent night shift schedule with six consecutive 12-hour work periods followed by 1 day off.

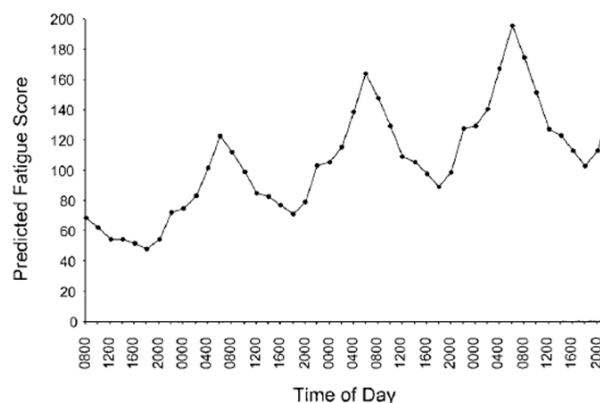


Figure 1. the simulation results using FAID.

Figure 1 is a simulation of 88 hours of work and rest. In this figure, the worker repeats a schedule of working 12 hours followed by 12 hours of rest. From the results, it can be seen that although the workload for each day is the same, the fatigue level builds up faster when the worker is in a higher fatigue state. Moreover, we also want the fatigue level of our model to vary with respect to the workload. From the above discussion, we choose an ordinary differential equation, $\dot{x} = kx$, as the mathematical fatigue model. From the slope aspect, the value k can be set according to the workload to describe the effects of different workloads on the level of fatigue. Also, the tendency of the function can be affected by the value x so that different fatigue states will have different effects on changing the fatigue level under the same workload. The optimal shift scheduling problem can be viewed as to decrease all fatigue states x by choosing an appropriate k which reflects the work/rest condition of each employee.

Assume the value of k is fixed during each time interval. Using the proposed model, the fatigue state can be formulated as

$$x_p = x_0 \exp\left(T \sum_{i=1}^p k_i\right), \quad (1)$$



in which x_p represents the fatigue state at time pT , and k_i is the exponential parameter in each time interval of length T . To compare our model with the FAID results, we take the time interval T as 2 hours and adjust appropriate values of k_i to fit the fatigue variation in each time interval during the first day. Assume k_i to be recurring each day. According to Figure 1, we set $[k_1 k_2 k_3 \dots k_{12}] = [-0.088 -0.121 -0.11 \dots 0.22]$ and use the same k_i for each day's schedule. $x_0 = 70$ is the initial value of the fatigue state according to Figure 1. Figure 2 is the result of a simulated fatigue curve using the proposed model. It can be seen that the proposed model does perform like the FAID model.

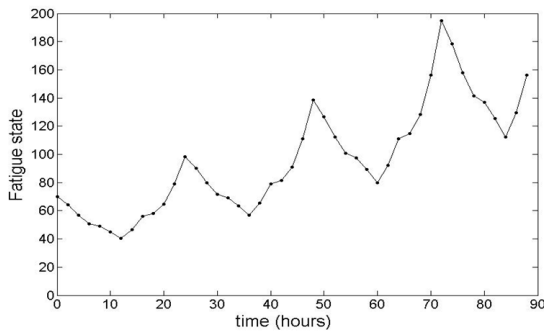


Figure 2. Fatigue curve generated from exponential functions.

Although the results of Figure 2 show promising results, it should be noted that the fatigue variation in the FAID model tends to stabilize when the fatigue state is beyond the extreme region. This phenomenon does not occur using the proposed model. However, this inconsistency occurs only when inappropriate shift scheduling is used so that the resultant fatigue state is beyond the tolerable range. Therefore this difference between the FAID model and the proposed model is considered acceptable.

Fatigue description model

We use variable $x_{i,j}$ to represent the i -th person's fatigue state at the j -th moment in which $j \in \{1, 2, 3, \dots, m\}$, and $i \in \{1, 2, 3, \dots, n\}$. Here, m is the number of time periods we consider, and n is the number of employees to be scheduled. The variable $k_{i,j}$ is used to represent the *fatigue coefficient*. If the i -th person at the j -th moment is on duty, $k_{i,j} > 0$. On the other hand, $k_{i,j} < 0$ if the i -th person is not working at the j -th moment. Different workloads can be set using different values of $k_{i,j}$. Since the perception of fatigue varies from person to person, we allow the possibility of assigning different fatigue coefficients among workers. We could also incorporate the fatigue increment for commuters if more precise estimation of the fatigue state is desired.

Optimal shift problem

In the optimal shift problem, the objective is to minimize the maximum fatigue value among all employees. That is, we need to minimize

$$J = \max\{x_{1,1}, x_{1,2}, x_{1,3}, \dots, x_{n,m}\}. \tag{2}$$

We use another variable Y as the new cost function. With the constraints $x_{i,j} < Y$ for all i, j , minimizing Y effectively minimizes J . The basic form of the optimal shift problem is described as the following:

$$\begin{aligned} \min_{k_{i,j}} \quad & Y \\ \text{s. t.} \quad & x_{i,j} \leq Y \\ & \text{for } i \in \{1, 2, \dots, n\}, j \in \{1, 2, \dots, m\} \end{aligned} \tag{3}$$

where $x_{i,j}$ can be written as

$$x_{i,j} = x_{i,0} \exp\left(T \sum_{l=1}^j k_{i,l}\right). \tag{4}$$

The problem is nonlinear in the decision variables, $k_{i,j}$. We can take the natural logarithms of (4) and let $Y_e = \ln(Y)$ to transform (4) into an equivalent linear form. The optimal shift problem becomes

$$\begin{aligned} \min_{k_{i,j}} \quad & Y_e \\ \text{s. t.} \quad & \ln x_{i,j} \leq Y_e \\ & \ln x_{i,j} = \ln x_{i,0} + T \sum_{l=1}^j k_{i,l} \\ & \text{for } i \in \{1, 2, \dots, n\}, j \in \{1, 2, \dots, m\} \end{aligned} \tag{5}$$

This optimization problem is now linear in the decision variables.

Shift property constraints

The shift property constraints include holidays, work hour rules and manpower requirements. To simplify the formulation of these constraints, we assume there are fixed types of shifts that employees can choose from. Then the optimization problem is to choose the best combination of shifts for each employee so that their maximum fatigue level can be minimized.

In order to use the idea of shift selection, we use the shift selection vector β , whose elements are defined as the following:

$$\beta = [\beta_{1,1,1} \ \beta_{1,1,2} \ \dots \ \beta_{1,1,s} \ \beta_{1,2,1} \ \dots \ \beta_{n,d,s}]. \tag{6}$$

The element $\beta_{i,j,k} \in \{0,1\}$ means whether the i -th ATC at the j -th day is assigned to the k -th shift or not. s

and d are the number of shift types and the number of days considered in the optimization problem. For example, if we assign the first employee who has to work the second shift on the third day, we set $\beta_{1,3,2} = 1$.

Let us define the shift work hour matrix W as

$$W = [w_1^T \quad w_2^T \quad w_3^T \quad \dots \quad w_s^T]^T, \quad (7)$$

where

$$w_i = [h_{i,1} \quad h_{i,2} \quad h_{i,3} \quad \dots \quad h_{i,\frac{24}{T}}], h_{i,j} \in \{0,1\}, \quad (8)$$

and $w_s = \mathbf{0}$. w_i is a vector representing the working hour distribution in the i -th shift. The elements of w_i are of binary values, which means the employee is either on duty or not at each time period. For example, if the work time in the first shift is from 0100 to 0800, and $T=1$, this shift is represented as $w_1 = [0 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ \dots \ 0]$. Using this formulation, W represents the working hour distribution of all shifts. The variable s is the number of different shifts under consideration. It should be noted that we place a zero vector, w_s , to represent a day off. That is, any employee assigned to the s -th shift on a specific day will not work the entire day.

Let α be the work state vector as

$$\alpha = [\alpha_{1,1} \quad \alpha_{1,2} \quad \dots \quad \alpha_{1,m} \quad \alpha_{2,1} \quad \dots \quad \alpha_{n,m}], \quad (9)$$

where $\alpha_{i,j} \in \{0,1\}$ means the i -th employee is either working or not working at the j -th time period. Then the relation between α and β can be represented by using

$$\alpha = \beta(I_{n,d} \otimes W). \quad (10)$$

Let k be the vector of the resultant fatigue coefficients as

$$k = [k_{1,1} \quad k_{1,2} \quad \dots \quad k_{1,m} \quad k_{2,1} \quad \dots \quad k_{n,m}]. \quad (11)$$

Then the relationships between $k_{i,j}$ and $\alpha_{i,j}$ can be shown as

$$\begin{aligned} k_{i,j} &= \alpha_{i,j} k_{i,j,w} + (\alpha_{i,j} - 1) k_{i,j,o} \\ k_{i,j,w} &\geq 0, k_{i,j,o} \geq 0 \end{aligned} \quad (12)$$

$k_{i,j,w}$ and $k_{i,j,o}$ are the coefficients of work and rest for the i -th employee at the j -th time period, respectively. These coefficients are work shift dependent and should be chosen based on real situations for each shift.

To assign more sophisticated fatigue coefficients for different work hours and different workers, we use the shift fatigue matrix

$$\tilde{W} = \begin{bmatrix} \tilde{W}_1 & 0 & \dots \\ 0 & \tilde{W}_2 & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}, \quad (13)$$

where \tilde{W}_j is the fatigue coefficient matrix for worker j .

Each row vector in \tilde{W}_j lists the fatigue coefficients for worker j when being assigned to the associated work shift. The fatigue coefficient vector k can then be computed by

$$k = \beta \tilde{W}. \quad (14)$$

The above variables will then be used to formulate the constraints relating to the shift properties. The shift property constraints can be divided into four categories.

Day off

Each employee's day off can be assigned according to his/her preferences before scheduling shifts. Let D_i be the set of the dates of the day off for the i -th employee. Then we set the associated $\beta_{i,q,s}$ according to the elements in D_i as

$$\beta_{i,q,s} = 1, \text{ for } \begin{cases} i \in \{1,2,3,\dots,n\} \\ q \in D_i \end{cases}. \quad (15)$$

These constraints ensure that the day off shifts have to be assigned to the dates listed in D_i .

Manpower requirement

The manpower requirements are regulated according to actual requirement and regulations. There are some special cases in these constraints. Take an 8-hour shift as an example. There are 5 possible kinds of shift schedules if the regulation permits a worker to work from 0000 to 0800 and from 1600 to 2400 in one day. The matrix W is represented as the following:

$$W = \begin{bmatrix} 1_{1 \times 8} & 0_{1 \times 8} & 0_{1 \times 8} \\ 0_{1 \times 8} & 1_{1 \times 8} & 0_{1 \times 8} \\ 0_{1 \times 8} & 0_{1 \times 8} & 1_{1 \times 8} \\ 1_{1 \times 8} & 0_{1 \times 8} & 1_{1 \times 8} \\ 0_{1 \times 8} & 0_{1 \times 8} & 0_{1 \times 8} \end{bmatrix}. \quad (16)$$

It can be seen that w_4 is the combination of w_1 and

w_3 . From the manpower requirement point of view, we must consider the contribution of w_4 to the first and third shifts. This means that there may be more than one row in W that has manpower contribution to a specific work position. Let $M_{j,k}$ be the number of the manpower requirements for the k-th shift on the j-th day. We set $\bar{\beta}_{j,k}$ to be a set whose components $\beta_{j,k,l}$ contain the possible shift schedules that have a contribution to the k-th shift on the j-th day. The manpower requirement constraints are then represented as the following:

$$\sum_{\beta_{i,j} \in \bar{\beta}_{j,k}} \beta_{i,j,l} = M_{j,k}, \text{ for } \begin{cases} k \in \{1,2,3,\dots,s\} \\ j \in \{1,2,3,\dots,d\} \end{cases} \quad (17)$$

Shift constraints

Shift constraints consider a situation in which each employee may or may not be assigned to some shifts together. For example, the company or government may regulate that employees cannot work two consecutive shifts or that employees cannot work on two different shifts at the same time. Since β contains all the allowable combinations of shifts in each day, for each fixed i and j , only one element of $\beta_{i,j,k} \ k \in \{1,\dots,s\}$ can have value of 1 while other elements should be restricted to being 0s. That is, each employee should be assigned to only one shift on each day regardless of whether it is a day off or a work day. This concept can be formulated as:

$$\sum_{l=1}^s \beta_{i,j,l} = 1, \text{ for } \begin{cases} i \in \{1,2,3,\dots,n\} \\ j \in \{1,2,3,\dots,d\} \end{cases} \quad (18)$$

where d and n represent the number of the simulation time in days and the number of employees, respectively. Other limitations, such as a night shift that may not be followed by an early shift can be formulated using similar ideas.

Work hour constraints

The government or company may regulate the maximum work hours in a day, a week, or a month. To formulate this constraint, we use the work hour vector

$$H = [h_{1,1,1} \ h_{1,1,2} \ \dots \ h_{1,1,s} \ h_{1,2,1} \ \dots \ h_{n,d,s}] \quad (19)$$

The value $h_{i,j,k}$ is the work hour corresponding to the shift $\beta_{i,j,k}$. To ensure the work hours for the i-th person between day d_1 and d_2 to be at most H_w hours, we use the following constraint:

$$\sum_{j=d_1}^{d_2} \left(\sum_{k=1}^s \beta_{i,j,k} h_{i,j,k} \right) \leq H_w \quad (20)$$

Shift scheduling problem definition

Combining the objective function and the above constraints, we get the following integer programming problem:

$$\begin{aligned} \min_{\beta} \quad & Y_e \\ \text{s.t.} \quad & \ln x_{i,0} + T \sum_{l=1}^t (\alpha_{i,l} k_{i,l,w} + (\alpha_{i,l} - 1) k_{i,l,o}) \leq Y_e \\ & \beta_{i,q,s} = 1 \\ & \sum_{\beta_{i,j,k} \in \bar{\beta}_{j,k}} \beta_{i,j,k} = M_{j,k} \\ & \sum_{l=1}^s \beta_{i,j,l} = 1 \\ & \sum_{j=d_1}^{d_2} \left(\sum_{k=1}^s \beta_{i,j,k} h_{i,j,k} \right) \leq H_w \\ & \alpha = \beta(l_{n,d} \otimes W) \\ & \beta_{i,j,k} \in \{0,1\} \end{aligned} \quad (21)$$

for $i \in \{1,2,\dots,n\}$, $j \in \{1,2,\dots,d\}$, $k \in \{1,2,\dots,s\}$, $t \in \{1,2,\dots,m\}$ and $q \in D_i$. The integer programming problem for using the shift fatigue matrix \tilde{W} can also be formulated in a similar way by using (14) and substituting the vector k into the first constraint.

ATC shift scheduling simulation

Problem descriptions

In order to apply the proposed scheduling algorithm, we need to know the values of $k_{i,j,w}$ and $k_{i,j,o}$. For simplicity, in the following simulations, we will use fixed values for $k_{i,j,w}$ and $k_{i,j,o}$. We use some assumptions to get a rough estimate of these coefficients. We assume that each ATC's fatigue variation is under a *stable situation*. A stable situation means an ATC's fatigue state will recover to approximately the initial state after experiencing a cycle of average work and rest hours. The ratio of the average work and rest time is calculated from a sample ATC schedule. This ratio is then applied to calculate the ratio of on duty coefficient, $k_{i,j,w}$, and off duty coefficient, $k_{i,j,o}$. These two coefficients are then adjusted to appropriate values so that with the initial fatigue value of 5, the fatigue state value reaches 40 after working for 8 hours.

Due to the limited number of available ATCs and their specific commuting requirements, there are eleven possible shift combinations. These combinations are

listed in Table 1. Because 0700 is the start of the work cycle, we set the time point 0700 to be the first column of W . In this simulation, we use $k_{i,j,w} = 0.1733$, and $k_{i,j,o} = 0.365$ respectively, according to the considerations of the *stable situation*.

The objective is to minimize the least upper bound of the fatigue state level. The constraints are listed as the following:

Table 1. Shift combinations.

| Type | Work Time | Type | Work Time |
|------|-----------|------|-----------------------|
| A | 0700-1300 | G | 1300-2200 |
| B | 0800-1300 | H | 1900-0700 |
| C | 0900-1900 | I | 0700-1300 & 1900-0700 |
| D | 1000-1900 | J | 0800-1300 & 1900-0700 |
| E | 1300-1900 | O | Day Off |
| F | 1300-2100 | | |

- From our selection of fatigue coefficients, the highest fatigue value occurs only at the end of a shift. Therefore, we only select the end of all possible shifts and make Y_e bigger than the fatigue value at those points to reduce the number of constraints of the least upper bound condition. We set the initial value $x_{i,o} = 5$, and $T = 1$ and bound the fatigue values at 0700, 1300, 1900, 2100, and 2200 to establish the constraints. This step helps us remove 800 constraints.

Table 2. The original shift table.

| | Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| ATC 1 | O | O | O | O | O | O | O |
| ATC 2 | I | O | A | C | O | C | I |
| ATC 3 | C | F | C | I | O | O | I |
| ATC 4 | H | O | O | A | F | C | I |
| ATC 5 | O | G | F | I | O | A | C |
| ATC 6 | B | C | O | B | G | O | O |

- The off-work dates in the shift simulation are identical to the ones in the original schedule shown in Table 2. We assign the employees to the 11-th shift during their days off. The associated day off sets are:

$$\begin{aligned}
 D_1 &= \{1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7\} & D_2 &= \{2 \ 5\} \\
 D_3 &= \{5 \ 6\} & D_4 &= \{2 \ 3\} \\
 D_5 &= \{1 \ 5\} & D_6 &= \{2 \ 3 \ 6 \ 7\}
 \end{aligned}
 \tag{22}$$

- From Table 1, the manpower requirement constraints are:

$$M_{j,k} = \begin{cases} \sum_{i=1}^6 (\beta_{i,j,1} + \beta_{i,j,9}) & , \text{for } k = 1 \\ \sum_{i=1}^6 (\beta_{i,j,2} + \beta_{i,j,10}) & , \text{for } k = 2 \\ \sum_{i=1}^6 (\beta_{i,j,8} + \beta_{i,j,9} + \beta_{i,j,10}) & , \text{for } k = 8 \\ \sum_{i=1}^6 \beta_{i,j,k} & , \text{otherwise} \end{cases}
 \tag{23}$$

for $j \in \{1, 2, \dots, 7\}$ and $k \in \{1, 2, \dots, 8\}$. Using (24) and the information from Table 2, $M_{j,k}$ can be represented as a vector form in the following:

$$\begin{aligned}
 M &= [M_{1,1} \ M_{1,2} \ M_{1,3} \ \dots \ M_{7,8}] \\
 &= [2 \ 1 \ 1 \ \dots \ 3]
 \end{aligned}
 \tag{24}$$

- Each ATC is assigned to one shift every day regardless of whether it is a day off or a work day. If there are too few ATCs available, and the manpower requirements are high, the problem may be infeasible.
- We take 60 hours as the maximum total work hours, which is larger than the general average work hours in a week. This is because in this small example, an ATC is on vacation the whole week, and other ATCs need work longer to satisfy the manpower requirements.

Simulation results

Using the proposed fatigue model and the original schedule table listed in Table 1, we get the fatigue variation for each ATC shown in Figure 3. The maximum fatigue state value is around 600. This peak value occurs at approximately 100 hours for ATC3. It can also be observed that the fatigue values of other ATCs are much lower. This means that the original shift table is not properly scheduled and should be improved to decrease the peak fatigue level.

Solving the optimization problem (21) with Ip_solve version 5.5, we get the optimal shift table listed in Table 3. The fatigue levels for each ATC are shown in Figure 4. From this figure, it can be seen that the maximum fatigue value has been decreased to 90 and that the fatigue values are much more evenly distributed among all ATCs.

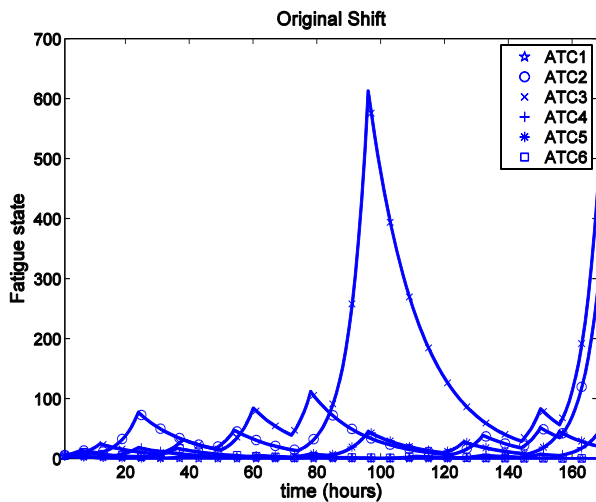


Figure 3. The fatigue levels using the original shift table.

Table 3. The optimal shift table.

| | Day 1 | Day 2 | Day 3 | Day 4 | Day 5 | Day 6 | Day 7 |
|-------|-------|-------|-------|-------|-------|-------|-------|
| ATC 1 | O | O | O | O | O | O | O |
| ATC 2 | H | O | C | A | O | C | I |
| ATC 3 | B | F | A | I | O | O | I |
| ATC 4 | I | O | O | A | F | A | I |
| ATC 5 | O | G | F | J | O | C | C |
| ATC 6 | C | C | O | C | G | O | O |

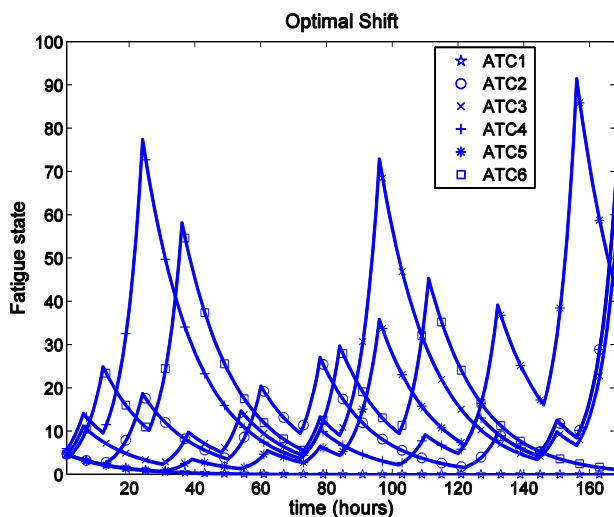


Figure 4. The fatigue levels using the optimal shift table.

The entire ATC group is a large group that contains 45 ATCs in our data. The usual shift table contains work shifts for a month. Here we will optimize the shift table for 45 ATCs during a period of 31 days. The number of variables is roughly 15,000, and there are 20,000 constraints to be satisfied. Figure 5 shows the results for the optimized shifts.

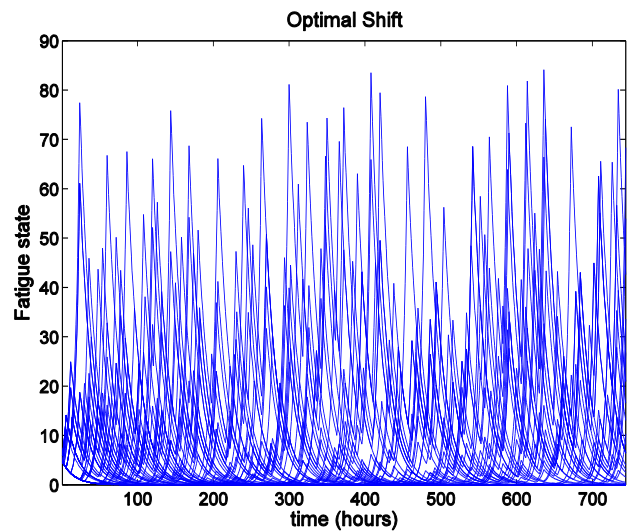


Figure 5. The fatigue levels using the optimal shift table of all ATCs.

Conclusions

In this paper, we apply integer programming to solve an ATC scheduling problem. The objective is to minimize the maximum fatigue in order to allow ATCs to avoid experiencing extreme fatigue. The fatigue variation for each employee is modeled by a first order differential equation. To establish the optimal problem, some constraints and assumptions, such as a combination of shift types and fatigue functions, are considered. Several examples are presented to show the power of the proposed approach.

In order to correctly predict the fatigue values, precise estimations of the work/rest coefficients are required. Several methods could potentially be used to obtain these coefficients, such as the use of questionnaires or medical instruments, or current available fatigue models such as FAID could be used to generate the fatigue variation curve then it could be used to estimate these coefficients. The estimation of these coefficients will be pursued in the near future.

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