ORIGINAL ARTICLE

Delay Dependent Robust H_∞ Filter design for Discrete Time-delay Systems with Missing Measurements via Homogeneous Polynomial Matrices

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(**Received** 4 February 2016; **Accepted** 25 March 2015; **Published on line** 1 September 2016)

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DOI: 10.5875/ausmt.v6i3.1112

Abstract: This paper presents new delay-dependent robust linear matrix inequality (LMI) conditions for a robust H_∞ filter for discrete time systems affected by time-varying state delays and missing measurements. Our attention is focused on the analysis and design of robust H_∞ filters. LMI relaxations based on homogeneous polynomial matrices of an arbitrary degree are used to determine the state-space realization of the filter. The missing measurements are described by a binary switching sequence satisfying a Bernoulli distribution. Numerical examples are presented to illustrate the effectiveness and applicability of the proposed method.

Keywords: robust h-infinity filtering, homogeneous polynomial matrices, missing measurements, time-delay, Linear Matrix Inequality (LMI), delay dependent condition

Introduction

Time delays are frequently encountered in many engineering systems and are usually regarded as a source of instability and poor performance. Recently, interest has grown in time-varying delayed systems in the field of networked control systems [1,2], and more attention has been paid to the stability of discrete time systems with delays. [3] The delay-dependent stability of discrete time systems with time varying delays [4] has improved results by choosing a new Lyapunov-Krasovskii function that provides less conservative results. Delay dependent stability robustness conditions for systems with two additive time delays are given in [5]. Similarly, the filtering problem for time-delay systems has been investigated by many authors in different contexts such as continuous systems [6-7], discrete-time systems [8-10] and neutral systems [11]. Filtering is an important issue in signal processing and control theory [12]. The H_∞ norm of the transfer function from the input noise to the output estimation error is among the most commonly used performance criteria. The problem becomes more complex when the model includes uncertainties, and a robust filter is needed. One popular approach is based on the existence of a common Lyapunov function for the entire domain of uncertainty, assuring the quadratic stability of the dynamic system associated with the estimation error and bounds to the values of the norm used as the performance criterion. Many studies provide quadratic stability based conditions for a robust filter in the form of linear matrix inequalities (LMIs) for continuous-time systems [13-17], discrete-time systems [18], and time-delay systems [19-20]. In [21-22], quadratic in the state Lyapunov functions with a polynomial dependence greater than one were used to improve the obtain optimal results [23]. Many previous studies also use scalar variables that need to be searched to provide better results with smaller guaranteed costs. Methods based on polynomially parameter-dependent Lyapunov functions are a natural extension of affine parameter-dependence to reduce the conservatism.

The aforementioned works implicitly assume the hypothesis of perfect measurements. Unfortunately, in many practical applications, such a hypothesis does not hold. For example, due to sensor temporal failures or network transmission delay/loss, at certain time points, the system measurements only contain noise, which indicates that the real signal is missing. The filtering problem for systems with missing measurements has received much attention in recent years. Fundamentally, there are two ways to model the missing measurement phenomena, i.e., using a binary switching sequence and using jump linear systems. The binary switching sequence is specified by a conditional probability distribution, it can be viewed as a Bernoulli distributed white sequence taking on values of 0 and 1 [24-28]. Another approach is to model the missing measurement as a Markovian jumping process, the filtering problem with missing measurements has been studied in [29,30], and filters guaranteeing expected estimation error covariance have been designed based on jump Riccati equations.

The robust H_∞ filtering for discrete-time systems with missing measurements has been addressed for several models, nonlinear systems [27,31], networked systems [32] and delayed systems [25,26,28].

However, it is worth mentioning that all of the H_∞ filtering results are concerned either with delay-dependent results for deterministic systems, delay independent results for systems with missing measurements, or delay-dependent results for switched systems with missing measurements. Currently, there is no robust H_∞ filter study for discrete time-delay systems with missing measurements. This article uses the polynomially parameter-dependent idea to solve the problem of robust H_∞ filtering for uncertain linear discrete systems with a time-varying delay and missing measurements.

This paper investigates the problem of robust H_∞ filter design for discrete-time linear systems affected by time-invariant polytopic uncertainty and a time-varying state delay. The main contribution is to provide new delay-dependent robust linear matrix inequality (LMI) conditions for the filter design. In addition, slack matrices are introduced to obtain Lyapunov matrices depending on uncertain parameters. Sufficient conditions for the existence of a feasible solution to the problem are derived, and expected filters can be determined in terms of LMIs. The approach is based on homogeneous polynomial matrices of arbitrary degree.

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Notation: Throughout this paper, the superscripts 'T' and '-1' respectively stand for the transpose and the inverse of a matrix; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $E\{x\}$ stands for the expectation of the stochastic variable x; Prob{.} means the occurrence probability of the event "."; $P > 0$ means that the matrix P is positive definite; I is an appropriately dimensioned identity matrix; diag{…} denotes a block-diagonal matrix; and the symmetric terms in a symmetric matrix are denoted by *

Problem Description and Preliminaries

Consider the following discrete time-delay system:

$$\begin{cases} x_{k+1} = A(\alpha)x_k + A_d(\alpha)x_{k-d_k} + B(\alpha)\omega_k, \\ z_k = C_0(\alpha)x_k + C_d(\alpha)x_{k-d_k} + D_0(\alpha)\omega_k, \\ \tilde{y}_k = C(\alpha)x_k + C_{dy}(\alpha)x_{k-d_k} + D(\alpha)\omega_k \\ x_k = \varphi_k, k = -d_M, -d_M + 1, \dots, 0, \end{cases} \quad (1)$$

where $x_k \epsilon \mathbb{R}^n$ is the state vector, $z_k \epsilon \mathbb{R}^m$ is the signal to be estimated, and $\omega_k \epsilon \mathbb{R}^p$ is the disturbance noise belonging to $l_2[0,\infty)$. φ_k is a real initial function on $[-d_M \;\; 0]$, and $A(\alpha), A_d(\alpha)$, $B(\alpha), C_0(\alpha)$, $C_d(\alpha)$, $D_0(\alpha)$, $C(\alpha), C_{dy}(\alpha)$, $D(\alpha)$ are unknown parameter matrices of appropriate dimensions, belonging to a polytopic domain parameterized in terms of a time-invariant vector α, being generally given by

$$Z(\alpha) = \sum_{i=1}^{N} \alpha_i Z_i, \alpha \in \Lambda_N \quad (2)$$

where $Z(\alpha)$ represents any matrix of the system in (1), $Z_i, i = 1, \dots, N$ are the vertices, N is the number of vertices of the polytope and Λ_N is the unit simplex, given by

$$\Lambda_N = \left\{ \alpha \; \epsilon \mathbb{R}^N : \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0, i = 1, \dots, N \right\} \quad (3)$$

The time delays are assumed to be time-varying and satisfy

$$d_m \leq d_k \leq d_M, \quad (4)$$

where d_m and d_M are nonnegative integers respectively denoting the lower and upper bounds of d_k.

Remark 1: The time delay d_k characterizes the real situation in many practical applications. A typical example containing time delays can be found in networked control systems, where the delays introduced by the network transmission (either from sensor to controller or from controller to actuator) are actually time-varying, and can be assumed to have lower and upper delay bounds without loss of generality.

In system (1), $\tilde{y}_k$ is the ideal system output. However, in practical engineering systems, the system output usually is imperfect with probabilistic missing data. Then, the obtained system output can be described by

$$y_k = \delta_k C(\alpha)x_k + \delta_k C_{dy}(\alpha)x_{k-d_k} + D(\alpha)\omega_k \qquad (5)$$

The stochastic variable δ_k is a Bernoulli distributed white sequence taking the values of 0 and 1 with:

$$\mathbf{Pr}\{\delta_k = 1\} = E\{\delta_k\} \coloneqq \beta, \qquad (6)$$
$$\mathbf{Pr}\{\delta_k = 0\} = 1 - E\{\delta_k\} \coloneqq 1 - \beta, \qquad (7)$$

Obviously, from condition (6), it yields

$$\mathrm{E}\{\delta_k - \beta\} = 0$$
$$E\{(\delta_k - \beta)^2\} \coloneqq \beta(1-\beta), \qquad (8)$$

The problem addressed in this paper is: find a full order robust linear filter given by

$$\begin{cases} \hat{x}_{k+1} = A_f\hat{x}_k + B_f y_k, \\ \hat{z}_k = C_f\hat{x}_k + D_f y_k, \end{cases} \qquad (9)$$

lter state vector, $\hat{z}_k$ is an estimate for z_k, A_f, B_f, C_f and D_f are filter parameters to be determined.
by defining the augmented vector

$$\eta_k \coloneqq \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} \qquad (10)$$

and using (1), (5) and (9), we obtain the following augmented system:

$$\begin{aligned} \eta_{k+1} &= \tilde{A}(\alpha)\eta_k + (\delta_k - \beta)\bar{A}(\alpha)\eta_k \\ &\quad + \tilde{A}_d(\alpha)\eta_{k-d_k} + (\delta_k - \beta)\bar{A}_d(\alpha)\eta_{k-d_k} \\ &\quad + \tilde{B}(\alpha)\omega_k, \\ \tilde{z} &\coloneqq z_k - \hat{z}_k \\ &= \tilde{C}(\alpha)\eta_k + (\delta_k - \beta)\bar{C}(\alpha)\eta_k + \tilde{C}_d(\alpha)\eta_{k-d_k} \\ &\quad + (\delta_k - \beta)\bar{C}_d(\alpha)\eta_{k-d_k} + \tilde{D}(\alpha)\omega_k, \end{aligned}$$

(11)
where

$$\tilde{A}(\alpha) \coloneqq \begin{bmatrix} A(\alpha) & 0 \\ \beta B_f C(\alpha) & A_f \end{bmatrix},\ \bar{A}(\alpha) \coloneqq \begin{bmatrix} 0 & 0 \\ B_f C(\alpha) & 0 \end{bmatrix},$$
$$\tilde{A}_d(\alpha) \coloneqq \begin{bmatrix} A_d(\alpha) & 0 \\ \beta B_f C_{dy}(\alpha) & 0 \end{bmatrix},\ \tilde{B}(\alpha) \coloneqq \begin{bmatrix} B(\alpha) \\ B_f D(\alpha) \end{bmatrix},$$
$$\bar{A}_d(\alpha) \coloneqq \begin{bmatrix} 0 & 0 \\ B_f C_{dy}(\alpha) & 0 \end{bmatrix},$$
$$\bar{C}(\alpha) \coloneqq \begin{bmatrix} -D_f C(\alpha) & 0 \end{bmatrix},$$
$$\tilde{C}(\alpha) \coloneqq \begin{bmatrix} C_0(\alpha) - \beta D_f C(\alpha) \text{-} C_f \end{bmatrix},$$
$$\tilde{C}_d(\alpha) \coloneqq \begin{bmatrix} C_d(\alpha) - \beta D_f C_{dy}(\alpha) & 0 \end{bmatrix},$$
$$\bar{C}_d(\alpha) \coloneqq \begin{bmatrix} -D_f C_{dy}(\alpha) & 0 \end{bmatrix},$$
$$\tilde{D}(\alpha) \coloneqq D_0(\alpha) - D_f D(\alpha)$$

(12)

Since the filtering error system (11) contains the stochastic variable δ_k, we introduce the definition of stochastic stability in the mean-square sense.

Definition 1: [25] the filtering error system (11) is said to be exponentially mean-square stable if, with $\omega_k = 0$, for any initial conditions, there exist constants $\sigma > 0$ and $\mu \in (0,1)$ such that

$$E\{|\eta_k|^2\} \le \sigma\mu^k \sup_{-d_M \le i \le 0} E\{|\eta_i|^2\},\ k \in \mathbb{Z}^+ \qquad (13)$$

Assumption 1: System (1) is exponentially mean-square stable for the whole uncertain domain (3).

Remark 2: Since the augmented state vector η_k of the filtering error system (11) is constituted by both x_k and $\hat{x}_k$, it follows that the error system (11) cannot be exponentially mean-square stable only if the system (1) is exponentially mean-square stable.

This paper seeks to design a filter (9) for the system (1) such that for all admissible parameter uncertainties satisfying (3) and all possible missing measurements (5), the filtering error system (11) is exponentially mean-square stable for $\omega_k = 0$ and for a prescribed scalar $\gamma > 0$, and under the zero-initial condition, the following inequalities holds

$$\sum_{k=0}^{\infty} E\{|\tilde{z}_k|^2\} \le \gamma^2 \sum_{k=0}^{\infty} E\{|\omega_k|^2\} \qquad (14)$$

for all nonzero ω_k.

Robust H_∞ Filtering Analysis

This section provides a robust H_∞ performance analysis for the filtering error system (11), which will be used for the filter design in the following section.

Theorem 1: the maximum energy gain from ω_k to $\tilde{z}_k$ is limited by γ and the augmented dynamic system (11) is exponentially mean-square stable for all $\alpha \in \Lambda_N$ if there exist parameter – dependent symmetric positive definite matrices $P(\alpha) \in \mathbb{R}^{2n\times 2n}, Q(\alpha) \in \mathbb{R}^{2n\times 2n}$, $Q_1(\alpha) \in \mathbb{R}^{2n\times 2n}, Q_2(\alpha) \in \mathbb{R}^{2n\times 2n}, R_1(\alpha) \in \mathbb{R}^{2n\times 2n}$ and $R_2(\alpha) \in \mathbb{R}^{2n\times 2n}$ such that

$$\begin{pmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ * & \Psi_{22} & 0 \\ * & * & -\gamma^2 I \end{pmatrix} < 0 \qquad (15)$$

where

$$\Psi_{12}=\begin{bmatrix} P(\alpha)\tilde{A}(\alpha) & P(\alpha)\tilde{A}_d(\alpha) & 0 & 0\\ \rho_2 P(\alpha)\bar{A}(\alpha) & \rho_2 P(\alpha)\bar{A}_d(\alpha) & 0 & 0\\ d_m R_1(\alpha)(\tilde{A}(\alpha)-I) & d_m R_1(\alpha)\tilde{A}_d(\alpha) & 0 & 0\\ d_m\rho_2 R_1(\alpha)\bar{A}(\alpha) & d_m\rho_2 R_1(\alpha)\bar{A}_d(\alpha) & 0 & 0\\ d_{Mm}R_2(\alpha)(\tilde{A}(\alpha)-I) & d_{Mm}R_2(\alpha)\tilde{A}_d(\alpha) & 0 & 0\\ d_{Mm}\rho_2 R_2(\alpha)\bar{A}(\alpha) & d_{Mm}\rho_2 R_2(\alpha)\bar{A}_d(\alpha) & 0 & 0\\ \tilde{C}(\alpha) & \tilde{C}_d(\alpha) & 0 & 0\\ \rho_2\bar{C}(\alpha) & \rho_2\bar{C}_d(\alpha) & 0 & 0\end{bmatrix}$$

$$\Psi_{11}=diag\{-P(\alpha),-P(\alpha),-R_1(\alpha),-R_1(\alpha),-R_2(\alpha),-R_2(\alpha),-I,-I\},$$

$$\Psi_{13}=[\tilde{B}^T(\alpha)P^T(\alpha)\quad 0\quad d_m\tilde{B}^T(\alpha)R_1^T(\alpha)\quad 0\quad d_m\tilde{B}^T(\alpha)R_2^T(\alpha)\quad 0\quad \tilde{D}^T(\alpha)\quad 0]^T,$$

$$\Psi_{22}=\begin{bmatrix} -P(\alpha)+\rho_1 Q(\alpha)+Q_1(\alpha)+Q_2(\alpha)-R_1(\alpha) & 0 & R_1(\alpha) & 0\\ * & -Q(\alpha) & 0 & 0\\ * & * & -Q_1(\alpha)-R_1(\alpha)-R_2(\alpha) & R_2(\alpha)\\ * & * & * & -Q_2(\alpha)-R_2(\alpha)\end{bmatrix},$$

$\rho_1=d_M-d_m+1$, $d_{Mm}=d_M-d_m$ and $\rho_2=\sqrt{\beta(1-\beta)}$.

Proof Choose a Lyapunov function candidate as

$$V(\alpha,\Theta_k)=V_1(\alpha,\Theta_k)+V_2(\alpha,\Theta_k)+V_3(\alpha,\Theta_k)+V_4(\alpha,\Theta_k)+V_5(\alpha,\Theta_k)+V_6(\alpha,\Theta_k)\qquad(16)$$

where $\Theta_k:=[\eta_k^T,\eta_{k-1}^T,\dots,\eta_0^T],\eta_k$ is defined in (10) and

$$V_1(\alpha,\theta_k)=\eta_k^T P(\alpha)\eta_k\qquad(17)$$

$$V_2(\alpha,\theta_k)=\sum_{i=k-d_k}^{k-1}\eta_i^T Q(\alpha)\eta_i\qquad(18)$$

$$V_3(\alpha,\theta_k)=\sum_{j=-d_M+2}^{-d_m+1}\sum_{i=k+j-1}^{k-1}\eta_i^T Q(\alpha)\eta_i\qquad(19)$$

$$V_4(\alpha,\theta_k)=\sum_{i=k-d_m}^{k-1}\eta_i^T Q_1(\alpha)\eta_i+\sum_{i=k-d_M}^{k-1}\eta_i^T Q_2(\alpha)\eta_i\qquad(20)$$

$$V_5(\alpha,\theta_k)=\sum_{i=-d_M}^{-1}\sum_{j=k+i}^{k-1}d_m\vartheta_j^T R_1(\alpha)\vartheta_j\qquad(21)$$

$$V_6(\alpha,\theta_k)=\sum_{i=-d_M}^{-d_m-1}\sum_{j=k+i}^{k-1}(d_M-d_m)\vartheta_j^T R_2(\alpha)\vartheta_j\qquad(22)$$

$$\vartheta_k=\eta_{k+1}-\eta_k\qquad(23)$$

Define $\Delta V_i(\alpha,\Theta_k)=E\{V_i(\alpha,\Theta_{k+1})|\Theta_k\}-E\{V_i(\alpha,\Theta_k)\}$.From (11), we can obtain the difference of the Lyapunov function with $\omega_k=0$ as follows:

$$\begin{aligned}\Delta V_1(\alpha,\Theta_k)&=\eta_{k+1}^T P(\alpha)\eta_{k+1}-\eta_k^T P(\alpha)\eta_k\\&=\eta_k^T[\tilde{A}^T(\alpha)P(\alpha)\tilde{A}(\alpha)+\beta(1-\beta)\bar{A}^T(\alpha)P(\alpha)\bar{A}(\alpha)\\&\quad-P(\alpha)]\ \eta_k+2\eta_k^T[\tilde{A}^T(\alpha)P(\alpha)\tilde{A}_d(\alpha)\\&\quad+\beta(1-\beta)\bar{A}^T(\alpha)P(\alpha)\bar{A}_d(\alpha)]\eta_{k-d_k}\\&\quad+\eta_{k-d_k}^T[\tilde{A}_d^T(\alpha)P(\alpha)\tilde{A}_d(\alpha)\\&\quad+\beta(1-\beta)\bar{A}_d^T(\alpha)P(\alpha)\bar{A}_d(\alpha)]\eta_{k-d_k}\end{aligned}$$

(24)

$$\begin{aligned}\Delta V_2(\alpha,\theta_k)&=\sum_{i=k+1-d_{k+1}}^{k}\eta_i^T Q(\alpha)\eta_i-\sum_{i=k-d_k}^{k-1}\eta_i^T Q(\alpha)\eta_i\\&=\eta_k^T Q(\alpha)\eta_k-\eta_{k-d_k}^T Q(\alpha)\eta_{k-d_k}+\sum_{i=k+1-d_{k+1}}^{k-1}\eta_i^T Q(\alpha)\eta_i-\\&\quad\sum_{i=k+1-d_k}^{k-1}\eta_i^T Q(\alpha)\eta_i\end{aligned}\qquad(25)$$

since

$$\sum_{i=k+1-d_{k+1}}^{k-1}\eta_i^T Q(\alpha)\eta_i=\sum_{i=k+1-d_{k+1}}^{k-d_m}\eta_i^T Q(\alpha)\eta_i+\sum_{i=k+1-d_m}^{k-1}\eta_i^T Q(\alpha)\eta_i\qquad(26)$$

we get

$$\sum_{i=k+1-d_{k+1}}^{k-1}\eta_i^T Q(\alpha)\eta_i\le\sum_{i=k+1-d_M}^{k-d_m}\eta_i^T Q(\alpha)\eta_i+\sum_{i=k+1-d_k}^{k-1}\eta_i^T Q(\alpha)\eta_i\qquad(27)$$

then

$$\Delta V_2(\alpha,\theta_k) \le \eta_k^T Q(\alpha)\eta_k - \eta_{k-d_k}^T Q(\alpha)\eta_{k-d_k} + \sum_{i=k+1-d_M}^{k-d_m} \eta_i^T Q(\alpha)\eta_i \tag{28}$$

$$\Delta V_3(\alpha,\theta_k) = \sum_{-d_M+2}^{-d_m+1}\sum_{i=k+j}^{k} \eta_i^T Q(\alpha)\eta_i - \sum_{-d_M+2}^{-d_m+1}\sum_{i=k+j-1}^{k-1} \eta_i^T Q(\alpha)\eta_i$$
$$= (d_M - d_m)\eta_k^T Q(\alpha)\eta_k - \sum_{i=k-d_{M+2}}^{k-d_m+1} \eta_i^T Q(\alpha)\eta_i \tag{29}$$

$$\Delta V_4(\alpha,\theta_k) = \eta_k^T Q_1(\alpha)\eta_k + \eta_k^T Q_2(\alpha)\eta_k$$
$$-\eta_{k-d_m}^T Q_1(\alpha)\eta_{k-d_m} - \eta_{k-d_M}^T Q_2(\alpha)\eta_{k-d_M}$$

$$\Delta V_4(\alpha,\theta_k) = \sum_{i=k+1-d_m}^{k} \eta_i^T Q_1(\alpha)\eta_i + \sum_{i=k+1-d_M}^{k} \eta_i^T Q_2(\alpha)\eta_i$$
$$-\sum_{i=k-d_m}^{k-1} \eta_i^T Q_1(\alpha)\eta_i + \sum_{i=k-d_M}^{k-1} \eta_i^T Q_2(\alpha)\eta_i \tag{30}$$

$$\Delta V_5(\alpha,\theta_k) = \sum_{i=-d_m}^{-1}\sum_{j=k+1+i}^{k} d_m \vartheta_j^T R_1(\alpha)\vartheta_j$$
$$-\sum_{i=-d_m}^{-1}\sum_{j=k+i}^{k-1} d_m \vartheta_j^T R_1(\alpha)\vartheta_j \tag{31}$$

$$= d_m^2 \vartheta_k^T R_1(\alpha)\vartheta_k - d_m \sum_{j=k-d_m}^{k-1} \vartheta_j^T R_1(\alpha)\vartheta_j \tag{32}$$

Using Jensen's Inequality we can write

$$-d_m \sum_{i=k-d_m}^{k-1} \vartheta_i^T R_1(\alpha)\vartheta_i \le -\left(\sum_{i=k-d_m}^{k-1} \vartheta_i\right)^T R_1(\alpha)\left(\sum_{i=k-d_m}^{k-1} \vartheta_i\right) \tag{33}$$

Then we obtain

$$\Delta V_5(\alpha,\theta_k) \le d_m^2 \eta_k^T [(\tilde{A}(\alpha)-I)^T R_1(\alpha)(\tilde{A}(\alpha)-I) +$$
$$\beta(1-\beta)\bar{A}^T(\alpha)R_1(\alpha)\bar{A}(\alpha)]\eta_k + 2\eta_k^T[(\tilde{A}(\alpha)-I)^T R_1(\alpha)\tilde{A}_d(\alpha)$$
$$+\beta(1-\beta)\bar{A}^T(\alpha)R_1(\alpha)\bar{A}_d(\alpha)]\eta_{k-d_k} + \eta_{k-d_k}^T[\tilde{A}_d^T(\alpha)R_1(\alpha)\tilde{A}_d(\alpha) +$$
$$\beta(1-\beta)\bar{A}_d^T(\alpha)R_1(\alpha)\bar{A}_d(\alpha)]\eta_{k-d_k} - \eta_k^T R_1(\alpha)\eta_k + 2\eta_k^T R_1(\alpha)\eta_{k-d_m}$$
$$-\eta_{k-d_m}^T R_1(\alpha)\eta_{k-d_m} \tag{34}$$

Following the same steps in (31)-(34) for computing $\Delta V_6(\alpha,\theta_k)$we obtain

$$\Delta V_6(\alpha,\theta_k) \le d_{Mm}^2 \eta_k^T [(A(\alpha)-I)^T R_2(\alpha)(A(\alpha)-I)$$
$$+\beta(1-\beta)\bar{A}^T(\alpha)R_2(\alpha)\bar{A}(\alpha)]\eta_k + 2\eta_k^T[(A(\alpha)-I)^T R_2(\alpha)A_d(\alpha)$$
$$+\beta(1-\beta)\bar{A}^T(\alpha)R_2(\alpha)\bar{A}_d(\alpha)]\eta_{k-d_k} + \eta_{k-d_k}^T[A_d^T(\alpha)R_2(\alpha)A_d(\alpha)$$
$$+\beta(1-\beta)\bar{A}_d^T(\alpha)R_2(\alpha)\bar{A}_d(\alpha)]\eta_{k-d_k} - \eta_{k-d_m}^T R_2(\alpha)\eta_{k-d_m}$$
$$+2\eta_{k-d_m}^T R_2(\alpha)\eta_{k-d_M} - \eta_{k-d_M}^T R_2(\alpha)\eta_{k-d_M} \tag{35}$$

finally

$$\Delta V(\alpha,\theta_k) = \sum_{i=1}^{6} \Delta V_i(\alpha,\theta_k)$$

then

$$\Delta V(\alpha,\theta_k) \le \xi_k^T \Xi \xi_k \tag{36}$$

where

$$\xi_k = \begin{bmatrix} \eta_k^T & \eta_{k-d_k}^T & \eta_{k-d_m}^T & \eta_{k-d_M}^T \end{bmatrix}^T, \tag{37}$$

$$\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 \\ * & \Xi_{22} & 0 & 0 \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{pmatrix}$$

where

$\Xi_{11} = \tilde{A}^T(\alpha)P(\alpha)\tilde{A}(\alpha) + \beta(1-\beta)\bar{A}^T(\alpha)[P(\alpha) + d_m^2 R_1(\alpha) + d_{Mm}^2 R_2(\alpha)]\bar{A}(\alpha) - P(\alpha) + \tau_1 Q(\alpha) + Q_1(\alpha) + Q_2(\alpha) + +(\tilde{A}(\alpha) - I)^T[d_m^2 R_1(\alpha) + d_{Mm}^2 R_2(\alpha)](\tilde{A}(\alpha) - I) - R_1(\alpha)$,
$\Xi_{12} = \tilde{A}^T(\alpha)P(\alpha)\tilde{A}_d(\alpha) + \beta(1-\beta)\bar{A}^T(\alpha)[P(\alpha) + d_m^2 R_1(\alpha) + d_{Mm}^2 R_2(\alpha)]\bar{A}_d(\alpha) + (\tilde{A}(\alpha) - I)^T[d_m^2 R_1(\alpha) + d_{Mm}^2 R_2(\alpha)]\tilde{A}_d(\alpha)$,$\Xi_{13} = R_1(\alpha)$,
$\Xi_{22} = \tilde{A}_d^T(\alpha)[P(\alpha) + d_m^2 R_1(\alpha) + d_{Mm}^2 R_2(\alpha)]\tilde{A}_d(\alpha) + \beta(1-\beta)\bar{A}_d^T(\alpha)[P(\alpha) + d_m^2 R_1(\alpha) + d_{Mm}^2 R_2(\alpha)]\bar{A}_d(\alpha) - Q(\alpha)$, $\Xi_{33} = -R_1(\alpha) - R_2(\alpha) - Q_1(\alpha)$,$\Xi_{34} = -R_2(\alpha)$,$\Xi_{44} = -R_2(\alpha) - Q_2(\alpha)$.

By Schur complement and simple manipulations, we arrive at the conclusion that (15) implies $\Xi < 0$. Therefore, for all nonzero η_k, we have $\Delta V(\alpha,\theta_k) < 0$, so we can find a positive scalar $\mu > 0$ such that

$$\Xi < \begin{bmatrix} -\mu I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (38)$$

and subsequently

$$E\{\Delta V(\alpha,\theta_k)\} < -\mu|\eta_k|^2. \quad (39)$$

From Lemma 1 of [25], we can confirm that the filtering error system (11) is exponentially mean-square stable.

Next, for any nonzero ω_k, it follows from (11) and (16-23) that

$$\begin{aligned} &E\{\Delta V(\alpha,\theta_k)\} + E\left\{\tilde{z}_k^T\tilde{z}_k\right\} - \gamma^2 E\{\omega_k^T\omega_k\} \le \\ &\xi_k^T \Xi \xi_k + (\tilde{C}(\alpha)\eta_k + (\delta_k-\beta)\bar{C}(\alpha)\eta_k + \\ &\tilde{C}_d(\alpha)\eta_{k-d_k} + (\delta_k-\beta)\bar{C}_d(\alpha)\eta_{k-d_k} + \\ &\tilde{D}(\alpha)\omega_k)^T(\tilde{C}(\alpha)\eta_k + (\delta_k-\beta)\bar{C}(\alpha)\eta_k + \\ &\tilde{C}_d(\alpha)\eta_{k-d_k} + (\delta_k-\beta)\bar{C}_d(\alpha)\eta_{k-d_k} + \tilde{D}(\alpha)\omega_k) - \\ &\gamma^2\omega_k^T\omega_k \\ &= E\{\xi_{1k}^T \Pi \xi_{1k}\} \end{aligned} \quad (40)$$

where

$\xi_{1k} = \begin{bmatrix} \eta_k^T & \eta_{k-d_k}^T & \eta_{k-d_m}^T & \eta_{k-d_M}^T & \omega_k^T \end{bmatrix}^T$,

and

$$\Pi = \begin{pmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & 0 & \Pi_{15} \\ * & \Pi_{22} & 0 & 0 & \Pi_{25} \\ * & * & \Pi_{33} & \Pi_{34} & 0 \\ * & * & * & \Pi_{44} & 0 \\ * & * & * & * & \Pi_{55} \end{pmatrix} \quad (41)$$

where

$\Pi_{11} = \Xi_{11} + \tilde{C}^T(\alpha)\tilde{C}(\alpha) + \beta(1-\beta)\bar{C}^T(\alpha)\bar{C}(\alpha)$,
$\Pi_{12} = \Xi_{12} + \tilde{C}^T(\alpha)\tilde{C}_d(\alpha) + \beta(1-\beta)\bar{C}^T(\alpha)\bar{C}_d(\alpha)$,
$\Pi_{13} = \Xi_{13}$,
$\Pi_{15} = \tilde{A}^T(\alpha)P(\alpha)B(\alpha) + \left(\tilde{A}(\alpha)-I\right)^T[d_m^2R_1(\alpha) + d_{Mm}^2R_2(\alpha)]B(\alpha) + \tilde{C}^T(\alpha)\tilde{D}(\alpha)$,
$\Pi_{22} = \Xi_{22} + \tilde{C}_d^T(\alpha)\tilde{C}_d(\alpha) + \beta(1-\beta)\bar{C}_d^T(\alpha)\bar{C}_d(\alpha)$,
$\Pi_{25} = \Xi_{25} + \tilde{A}_d^T(\alpha)[P(\alpha) + d_m^2R_1(\alpha) + d_{Mm}^2R_2(\alpha)]\tilde{B}(\alpha) + \tilde{C}_d^T(\alpha)\tilde{D}(\alpha)$, $\Pi_{33} = \Xi_{33}$,
$\Pi_{34} = \Xi_{34}$, $\Pi_{44} = \Xi_{44}$,
$\Pi_{55} = \tilde{B}^T(\alpha)[P(\alpha) + d_m^2R_1(\alpha) + d_{Mm}^2R_2(\alpha)]\tilde{B}(\alpha) + \tilde{D}^T(\alpha)\tilde{D}(\alpha) - \gamma^2 I$.

From (15), by applying Schur complement, it can be easily seen that $\Pi < 0$, and consequently

$$E\{\Delta V(\alpha,\theta_k)\} + E\left\{\tilde{z}_k^T\tilde{z}_k\right\} - \gamma^2 E\{\omega_k^T\omega_k\} < 0 \quad (42)$$

Now summing (42) from 0 to ∞ with respect to k yields

$$\sum_{k=0}^{\infty} E\{\|\tilde{z}_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{\|\omega_k\|^2\} + E\{V_0\} - E\{V_\infty\} \quad (43)$$

Since the system (11) is asymptotically mean-square stable for $\alpha \in \Lambda_N$, it is not difficult to see that the following inequality holds under the zero initial condition for all $\omega_k \in l_2$, which completes the proof.

$$\sum_{k=0}^{\infty} E\{\|\tilde{z}_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{\|\omega_k\|^2\} \quad (44)$$

Remark 3: The problem of H_∞ filtering for deterministic delayed systems via homogeneous polynomial matrices has been widely investigated with delay dependent results (see, for instance [27,31-32]). For delayed systems with missing measurements, this problem has been investigated in [23] but regardless of delay. In this paper, delay dependent H_∞ filtering for discrete systems via homogeneous polynomial matrices and missing measurements is investigated, by taking $\beta = 1$ (**Pr**$\{\delta_k = 1\}$) the system becomes deterministic which make our study more general.

Theorem 2: The maximum energy gain from ω_k to $\tilde{z}_k$ is limited by γ and the augmented dynamic system (11) is exponentially mean-square stable for all $\alpha \in \Lambda_N$ if there exist parameter – dependent symmetric positive definite matrices $P(\alpha) \in \mathbb{R}^{2n\times 2n}$, $Q(\alpha) \in \mathbb{R}^{2n\times 2n}$, $Q_1(\alpha) \in \mathbb{R}^{2n\times 2n}$, $Q_2(\alpha) \in \mathbb{R}^{2n\times 2n}$, $R_1(\alpha) \in \mathbb{R}^{2n\times 2n}$, $R_2(\alpha) \in \mathbb{R}^{2n\times 2n}$ and matrices $M(\alpha) \in \mathbb{R}^{2n\times 2n}$, $N_1(\alpha) \in \mathbb{R}^{2n\times 2n}$ and $N_2(\alpha) \in \mathbb{R}^{2n\times 2n}$ such that

$$\begin{pmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ * & \chi_{22} & 0 \\ * & * & -\gamma^2 I \end{pmatrix} < 0 \quad (45)$$

where

$\chi_{11} = diag\{\phi(\alpha), \phi(\alpha), \phi_1(\alpha), \phi_1(\alpha), \phi_2(\alpha), \phi_2(\alpha), -I, -I\}$,

$$\chi_{12} = \begin{bmatrix} M^T(\alpha)\tilde{A}(\alpha) & M^T(\alpha)\tilde{A}_d(\alpha) & 0 & 0 \\ \rho_2 M^T(\alpha)\bar{A}(\alpha) & \rho_2 M^T(\alpha)\bar{A}_d(\alpha) & 0 & 0 \\ d_m N_1^T(\alpha)(\tilde{A}(\alpha)-I) & d_m N_1^T(\alpha)\tilde{A}_d(\alpha) & 0 & 0 \\ d_m\rho_2 N_1^T(\alpha)\bar{A}(\alpha) & d_m\rho_2 N_1^T(\alpha)\bar{A}_d(\alpha) & 0 & 0 \\ d_{Mm} N_2^T(\alpha)(\tilde{A}(\alpha)-I) & d_{Mm} N_2^T(\alpha)\tilde{A}_d(\alpha) & 0 & 0 \\ d_{Mm}\rho_2 N_2^T(\alpha)\bar{A}(\alpha) & d_{Mm}\rho_2 N_2^T(\alpha)\bar{A}_d(\alpha) & 0 & 0 \\ \tilde{C}(\alpha) & \tilde{C}_d(\alpha) & 0 & 0 \\ \rho_2\bar{C}(\alpha) & \rho_2\bar{C}_d(\alpha) & 0 & 0 \end{bmatrix}$$

$$\chi_{13} = [\tilde{B}^T(\alpha)M(\alpha) \quad 0 \quad d_m\tilde{B}^T(\alpha)N_1(\alpha) \quad 0 \quad d_m\tilde{B}^T(\alpha)N_2(\alpha) \quad 0 \quad \tilde{D}^T(\alpha) \quad 0]^T$$

$$\chi_{22} = \begin{bmatrix} -P(\alpha)+\rho_1 Q(\alpha)+Q_1(\alpha)+Q_2(\alpha)-R_1(\alpha) & 0 & R_1(\alpha) & 0 \\ * & -Q(\alpha) & 0 & 0 \\ * & * & -Q_1(\alpha)-R_1(\alpha)-R_2(\alpha) & R_2(\alpha) \\ * & * & * & -Q_2(\alpha)-R_2(\alpha) \end{bmatrix}$$

$\phi = P(\alpha) - M(\alpha) - M^T(\alpha)$, $\phi_1 = R_1(\alpha) - N_1(\alpha) - N_1^T(\alpha)$, $\phi_2 = R_2(\alpha) - N_2(\alpha) - N_2^T(\alpha)$, $\rho_1 = d_M - d_m + 1$, $d_{Mm} = d_M - d_m$ and $\rho_2 = \sqrt{\beta(1-\beta)}$.

Proof: From (45), it can be seen that $M^T(\alpha) + M(\alpha) - P(\alpha) > 0$, $N_1^T(\alpha) + N_1(\alpha) - R_1(\alpha) > 0$ and $N_2^T(\alpha) + N_2(\alpha) - R_2(\alpha) > 0$. Since $P(\alpha), R_1(\alpha)$ and $R_2(\alpha)$ are positive definite, it follows that $M(\alpha), N_1(\alpha)$ and $N_2(\alpha)$ are nonsingular.. From $(M(\alpha) - P(\alpha))^T P^{-1}(\alpha)(M(\alpha) - P(\alpha)) \geq 0$, $(N_1(\alpha) - R_1(\alpha))^T R_1^{-1}(\alpha)(N_1(\alpha) - R_1(\alpha)) \geq 0$ and $(N_2(\alpha) - R_2(\alpha))^T R_2^{-1}(\alpha)(N_2(\alpha) - R_2(\alpha)) \geq 0$ we get $M^T(\alpha)P^{-1}(\alpha)M(\alpha) \geq M(\alpha) + M^T(\alpha) - P(\alpha)$, $N_1^T(\alpha)R_1^{-1}(\alpha)N_1(\alpha) \geq N_1(\alpha) + N_1^T(\alpha) - R_1(\alpha)$, and $N_2^T(\alpha)R_2^{-1}(\alpha)N_2(\alpha) \geq N_2(\alpha) + N_2^T(\alpha) - R_2(\alpha)$, together with (45), so we obtain

$$\begin{pmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ * & \chi_{22} & 0 \\ * & * & -\gamma^2 I \end{pmatrix} < 0 \qquad (46)$$

where

$\bar{\chi}_{11} = diag\{\bar{\phi}(\alpha), \bar{\phi}(\alpha), \bar{\phi}_1(\alpha), \bar{\phi}_1(\alpha), \bar{\phi}_2(\alpha), \bar{\phi}_2(\alpha), -I, -I\}$, $\bar{\chi}_{12} = \chi_{12}$, $\bar{\chi}_{13} = \chi_{13}$, $\bar{\chi}_{22} = \chi_{22}$, $\bar{\phi}(\alpha) := -M^T(\alpha)P^{-1}(\alpha)M(\alpha)$, $\bar{\phi}_2(\alpha) := -N_1^T(\alpha)R_1^{-1}(\alpha)N_1(\alpha)$ and $\bar{\phi}_2(\alpha) := -N_2^T(\alpha)R_2^{-1}(\alpha)N_2(\alpha)$.

Pré- and post-multiplying (46) by diag $\{P(\alpha)M^{-T}(\alpha), P(\alpha)M^{-T}(\alpha), R_1(\alpha)N_1^{-T}(\alpha), R_1(\alpha)N_1^{-T}(\alpha), R_2(\alpha)N_2^{-T}(\alpha), R_2(\alpha)N_2^{-T}(\alpha), I, I, I, I, I\}$ and its transpose, we obtain (15). This concludes the proof.

Remark 4: By introducing new additional matrices M, N_1 and N_2, which are not constrained to be symmetric or positive definite, we obtain a parameter dependent linear matrix inequality in which the Lyapunov matrices P, Q, Q_1, Q_2, R_1 and R_2 are not involved in any product with the dynamic matrices. If we require that $M = P$, $N_1 = R_1$ and $N_2 = R_2$, (45) reduces to (15), which means that Theorem 2 is less conservative than Theorem 1. Moreover, when $R = R_1 = R_2 = 0$, $Q_1 = Q_2 = 0$, the conditions of Theorem 1 and Theorem 2 reduce respectively to those of Lemmas 1 and 2 in [26]. It thus can be easily seen that Lemma 2 (or Lemma 1) in [26] is a special case of Theorem 1 (or Theorem 2). Note that the result of [26] is independent of delay. Consequently, the conditions of Theorems 1 and 2 are less conservative than those in Lemmas 1 and 2 in [26].

Robust H_∞ Filtering Synthesis

Theorems 1 and 2 have been established assuming the filter matrices is known. If this is not the case, it is a non-linear condition since the decision variables of interest (i.e. A_f, B_f, C_f and D_f) appear in sub-matrices multiplying other matrix variables. To derive tractable conditions for the filter design, structural constraints are imposed on the parameter-dependent matrices $M(\alpha), N_1(\alpha)$ and $N_2(\alpha)$as follows

$$M(\alpha) = \begin{bmatrix} M_1(\alpha) & M_2(\alpha) \\ K & K \end{bmatrix}$$
$$N_1(\alpha) = \begin{bmatrix} N_1(\alpha) & N_{12}(\alpha) \\ \lambda_1 K & \lambda_2 K \end{bmatrix}$$
$$N_2(\alpha) = \begin{bmatrix} N_2(\alpha) & N_{21}(\alpha) \\ \lambda_3 K & \lambda_4 K \end{bmatrix} \qquad (47)$$

where $K \in \mathbb{R}^{n\times n}$ and $\lambda_i, i = 1,2,3,4 \in \mathbb{R}$ are variables to be determined. For convenience matrices $P(\alpha)$, $Q(\alpha)$, $Q_1(\alpha)$, $Q_2(\alpha), R_1(\alpha)$ and $R_2(\alpha)$ are also partitioned in $n \times n$ blocks

$$P(\alpha) = \begin{bmatrix} P_{11}(\alpha) & P_{12}(\alpha) \\ * & P_{22}(\alpha) \end{bmatrix},$$
$$Q(\alpha) = \begin{bmatrix} Q_1(\alpha) & Q_2(\alpha) \\ * & Q_3(\alpha) \end{bmatrix},$$
$$Q_1(\alpha) = \begin{bmatrix} Q_{11}(\alpha) & Q_{112}(\alpha) \\ * & Q_{122}(\alpha) \end{bmatrix},$$
$$Q_2(\alpha) = \begin{bmatrix} Q_{21}(\alpha) & Q_{212}(\alpha) \\ * & Q_{222}(\alpha) \end{bmatrix},$$
$$R_1(\alpha) = \begin{bmatrix} R_{11}(\alpha) & R_{112}(\alpha) \\ * & R_{122}(\alpha) \end{bmatrix}, and$$
$$R_2(\alpha) = \begin{bmatrix} R_{21}(\alpha) & R_{212}(\alpha) \\ * & R_{222}(\alpha) \end{bmatrix} \qquad (48)$$

Theorem 3: If there exist symmetric parameter-dependent positive-definite matrices $P(\alpha), Q(\alpha), Q_1(\alpha), Q_2(\alpha), R_1(\alpha)$ and $R_2(\alpha)$ as in (48), matrices $M(\alpha)$, $N_1(\alpha)$ and $N_2(\alpha)$ as in (47), $\bar{A}_f, \bar{B}_f, C_f$ and D_f such that (49) holds for all $\alpha \in \Lambda_N$, then the performance bound given by γ and the exponential stability of the augmented dynamic system in

(11) are assured for all $\alpha \in \Lambda_N$. In this case, the desired filter of the form (9) is obtained with the following matrices:

$$B_f = K^{-T}\bar{B}_f,\ A_f = K^{-T}\bar{A}_f,\ C_f \text{ and } D_f. \tag{49}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{50}$$

where

$$\Omega_{11} = diag\{\phi(\alpha), \phi(\alpha), \phi_1(\alpha), \phi_1(\alpha), \phi_2(\alpha), \phi_2(\alpha), -I, -I\}$$

$$\phi(\alpha) = \begin{bmatrix} P_{11}(\alpha) - M_1(\alpha) - M_1^T(\alpha) & P_{12}(\alpha) - M_2(\alpha) - K^T(\alpha) \\ * & P_{22}(\alpha) - K(\alpha) - K^T(\alpha) \end{bmatrix}$$

$$\phi_1(\alpha) = \begin{bmatrix} R_{111}(\alpha) - N_1(\alpha) - N_1^T(\alpha) & R_{112}(\alpha) - N_{12}(\alpha) - \lambda_1 K^T \\ * & R_{122}(\alpha) - \lambda_2 K - \lambda_2 K^T \end{bmatrix}$$

$$\phi_2(\alpha) = \begin{bmatrix} R_{211}(\alpha) - N_2(\alpha) - N_2^T(\alpha) & R_{212}(\alpha) - N_{21}(\alpha) - \lambda_3 K^T \\ * & R_{222}(\alpha) - \lambda_4 K - \lambda_4 K^T \end{bmatrix}$$

$$\Omega_{12} = \begin{bmatrix}
\Upsilon_{115} & \bar{A}_f & \Upsilon_{117} & 0 & 0 & 0 & 0 & 0 \\
\Upsilon_{215} & \bar{A}_f & \Upsilon_{217} & 0 & 0 & 0 & 0 & 0 \\
\rho_2 \bar{B}_f C(\alpha) & 0 & \rho_2 \bar{B}_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
\rho_2 \bar{B}_f C(\alpha) & 0 & \rho_2 \bar{B}_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
\Upsilon_{515} & \Upsilon_{516} & \Upsilon_{517} & 0 & 0 & 0 & 0 & 0 \\
\Upsilon_{615} & \Upsilon_{616} & \Upsilon_{617} & 0 & 0 & 0 & 0 & 0 \\
\rho_2 d_m \lambda_1 \bar{B}_f C(\alpha) & 0 & \rho_2 d_m \lambda_1 \bar{B}_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
\rho_2 d_m \lambda_2 \bar{B}_f C(\alpha) & 0 & \rho_2 d_m \lambda_2 \bar{B}_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
\Upsilon_{915} & \Upsilon_{916} & \Upsilon_{917} & 0 & 0 & 0 & 0 & 0 \\
\Upsilon_{1015} & \Upsilon_{1016} & \Upsilon_{1017} & 0 & 0 & 0 & 0 & 0 \\
\rho_2 d_{Mm} \lambda_3 \bar{B}_f C(\alpha) & 0 & \rho_2 d_{Mm} \lambda_3 \bar{B}_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
\rho_2 d_{Mm} \lambda_4 \bar{B}_f C(\alpha) & 0 & \rho_2 d_m \lambda_4 \bar{B}_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
C_0(\alpha) - \beta D_f C(\alpha) & -C_f & C_d(\alpha) - \beta D_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0 \\
-\rho_2 D_f C(\alpha) & 0 & -D_f C_{dy}(\alpha) & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\Omega_{13} = [\Upsilon_{117}^T \quad \Upsilon_{217}^T \quad 0 \quad 0 \quad \Upsilon_{517}^T \quad \Upsilon_{617}^T \quad 0 \quad 0 \quad \Upsilon_{917}^T \quad \Upsilon_{1017}^T \quad 0 \quad 0 \quad \Upsilon_{1317}^T \quad 0]^T,$$

$$\Omega_{22} = \begin{bmatrix}
\Upsilon_{1515} & \Upsilon_{1516} & 0 & 0 & R_{111}(\alpha) & R_{112}(\alpha) & 0 & 0 \\
* & \Upsilon_{1616} & 0 & 0 & R_{112}^T(\alpha) & R_{122}(\alpha) & 0 & 0 \\
* & * & -Q_1(\alpha) & -Q_2(\alpha) & 0 & 0 & 0 & 0 \\
* & * & * & -Q_3(\alpha) & 0 & 0 & 0 & 0 \\
* & * & * & * & \Upsilon_{1919} & \Upsilon_{1920} & R_{211}(\alpha) & R_{212}(\alpha) \\
* & * & * & * & * & \Upsilon_{2020} & R_{212}^T(\alpha) & R_{222}(\alpha) \\
* & * & * & * & * & * & \Upsilon_{2121} & \Upsilon_{2122} \\
* & * & * & * & * & * & * & \Upsilon_{2222}
\end{bmatrix},$$

where

$\Upsilon_{115} = M_1^T(\alpha)A(\alpha) + \beta\bar{B}_f C(\alpha)$,

$\Upsilon_{117} = \rho_2 M_1^T(\alpha)A_d(\alpha) + \rho_2\beta\bar{\bar{B}}_f C_{dy}(\alpha)$,

$\Upsilon_{215} = M_2^T(\alpha)A(\alpha) + \beta\bar{\bar{B}}_f C(\alpha)$,

$\Upsilon_{217} = \rho_2 M_2^T(\alpha)A_d(\alpha) + \rho_2\beta\bar{\bar{B}}_f C_{dy}(\alpha)$,

$\Upsilon_{515} = d_m N_1^T(\alpha)(A(\alpha) - I) + d_m\lambda_1\beta\bar{B}_f C(\alpha)$, $\Upsilon_{516} = d_m\lambda_1\bar{\bar{A}}_f - d_m\lambda_1 K^T$

$\Upsilon_{517} = d_m N_1^T(\alpha)A_d(\alpha) + d_m\beta\lambda_1\bar{\bar{B}}_f C_{dy}(\alpha)$,

$\Upsilon_{615} = d_m N_{12}^T(\alpha)(A(\alpha) - I) + d_m\lambda_2\beta\bar{B}_f C(\alpha)$, $\Upsilon_{616} = d_m\lambda_2\bar{A}_f - d_m\lambda_2 K^T$

$\Upsilon_{617} = d_m N_{12}^T(\alpha)A_d(\alpha) + d_m\beta\lambda_2\bar{\bar{B}}_f C_{dy}(\alpha)$,

$\Upsilon_{915} = d_{Mm} N_2^T(\alpha)(A(\alpha) - I) + d_{Mm}\lambda_3\beta\bar{B}_f C(\alpha)$,

$\Upsilon_{916} = d_{Mm}\lambda_3\bar{\bar{A}}_f - d_{Mm}\lambda_3 K^T$

$\Upsilon_{917} = d_{Mm} N_2^T(\alpha)A_d(\alpha) + d_{Mm}\beta\lambda_3\bar{\bar{B}}_f C_{dy}(\alpha)$,

$\Upsilon_{1015} = d_{Mm} N_{21}^T(\alpha)(A(\alpha) - I) + d_{Mm}\lambda_4\beta\bar{B}_f C(\alpha)$,

$\Upsilon_{1016} = d_{Mm}\lambda_4\bar{A}_f - d_{Mm}\lambda_4 K^T$

$\Upsilon_{1017} = d_{Mm} N_{21}^T(\alpha)A_d(\alpha) + d_{Mm}\beta\lambda_4\bar{\bar{B}}_f C_{dy}(\alpha)$,

$\Upsilon_{117} = M_1^T(\alpha)B(\alpha) + \bar{B}_f D(\alpha)$,

$\Upsilon_{217} = M_2^T(\alpha)B(\alpha) + \bar{B}_f D(\alpha)$,

$\Upsilon_{517} = d_m N_1^T(\alpha)B(\alpha) + d_m\lambda_1\bar{B}_f D(\alpha)$,

$\Upsilon_{617} = d_m N_{12}^T(\alpha)B(\alpha) + d_m\lambda_2\bar{B}_f D(\alpha)$,

$\Upsilon_{917} = d_{Mm} N_2^T(\alpha)B(\alpha) + d_{Mm}\lambda_3\bar{B}_f D(\alpha)$,

$\Upsilon_{1017} = d_{Mm} N_{21}^T(\alpha)B(\alpha) + d_{Mm}\lambda_4\bar{B}_f D(\alpha)$,

$\Upsilon_{1317} = D_0(\alpha) - D_f D(\alpha)$,

$\Upsilon_{1515} = -P_{11}(\alpha) + \rho_1 Q_1(\alpha) + Q_{111}(\alpha) + Q_{211}(\alpha) - R_{111}(\alpha)$,

$\Upsilon_{1516} = -P_{12}(\alpha) + \rho_1 Q_2(\alpha) + Q_{112}(\alpha) + Q_{212}(\alpha) - R_{112}(\alpha)$,

$\Upsilon_{1616} = -P_{22}(\alpha) + \rho_1 Q_3(\alpha) + Q_{122}(\alpha) + Q_{222}(\alpha) - R_{122}(\alpha)$,

$\Upsilon_{1919} = -Q_{111}(\alpha) - R_{111}(\alpha) - R_{211}(\alpha)$,

$\Upsilon_{1920} = -Q_{112}(\alpha) - R_{112}(\alpha) - R_{212}(\alpha)$,

$\Upsilon_{2020} = -Q_{122}(\alpha) - R_{122}(\alpha) - R_{222}(\alpha)$,

$\Upsilon_{2121} = -Q_{211}(\alpha) - R_{211}(\alpha)$,

$\Upsilon_{2122} = -Q_{212}(\alpha) - R_{212}(\alpha)$,

$\Upsilon_{2222} = -Q_{222}(\alpha) - R_{222}(\alpha)$.

Proof: Considering the structure of slack variables $M(\alpha)$, $N_1(\alpha)$ and $N_2(\alpha)$ in (47) and matrices $P(\alpha), Q(\alpha)$, $Q_1(\alpha)$, $Q_2(\alpha)$, $R_1(\alpha)$ and $R_2(\alpha)$ in (48) and Theorem 2, the proof of Theorem 3 can be deduced easily, and is omitted here.

Remark 5: Note that with our approach, from Theorem 3, the number of decision variables required is $29n^2 + 6n + 4$, which is much smaller than $27n^2 + 7n + 10$ in [10]. The computational advantage is quite obvious.

Remark 6: When the scalars λ_1, λ_2, λ_3 and λ_4

are fixed, condition (50) becomes an LMI. However, choosing arbitrary λ_1, λ_2, λ_3 and λ_4 generally doesn't lead to optimal results. To simplify the test and obtain the best values of λ_1, λ_2, λ_3 and λ_4, we propose a tuning procedure for these parameters. With this procedure we search for the values λ_1, λ_2, λ_3 and λ_4 that ensure for a fixed value of lower and upper bounds of the delay, the minimal γ guarantees exponential mean square stability with H_∞ norm bound below a prescribed limit. Choose a cost function to be $f(\lambda_i) = \gamma_{opt}$ $i = 1..4$, for which (50) is true. The parameter γ_{opt} is obtained by solving the feasibility problem with the solver feasp in the LMI Toolbox. It is positive when there is no feasible solution to the set of LMIs under consideration. Finally apply a numerical optimization algorithm such as program *fminsearch* in the optimization toolbox to $f(\lambda_i) = \gamma_{opt}$ $i = 1..4$, under the constraint (50) to obtain a locally convergent solution to the problem. If the resulting minimum value of the cost function is negative, then the tuning parameter that solves the problem is found. This procedure can be derived as follows:

Algorithm (finding λ_i $i = 1..4$ that minimize γ)

- Step 1. Set an initial value for $[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$, fix d_m and d_M
- Step 2. If (48) is not feasible, return to step 1. Otherwise go to step 3
- Step 3. Solve the following problem: *min* (γ) subject to (50). Using the function *fminsearch* with $[\lambda_1, \lambda_2, \lambda_3, \lambda_4]_{init}$ and d_m and d_M and obtain the optimal value for $[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$.
- Step 4. Reinject the optimal values of $[\lambda_1, \lambda_2, \lambda_3, \lambda_4]$ on (50) to obtain a local minimum γ and suitable filter matrices.

To solve the robust LMI conditions of Theorem 3, the technique proposed in [33] to handle robust LMIs with parameters in the unit simplex can be applied. To this end, the polynomial matrices $(\alpha), Q(\alpha), Q_1(\alpha)$, $Q_2(\alpha)$, $R_1(\alpha)$, $R_2(\alpha), M(\alpha)$ $N_1(\alpha)$ and $N_2(\alpha)$ are treated as homogeneous polynomials of an arbitrary degree g, although different degrees can be used to produce results with distinct levels of complexity and accuracy. Let $Z_g(\alpha)$ be any parameter-dependent variable in (47) and (48) of an arbitrary degree g, denoted by

$$Z_g(\alpha) = \sum_{k \in \mathcal{K}(g)} \alpha_1^{k_1} \ldots \alpha_N^{k_N} Z_k, \quad k = k_1 k_2 \ldots k_N \tag{51}$$

where $\alpha_1^{k_1}\alpha_2^{k_2}\ldots\alpha_N^{k_N}, \alpha \in \Lambda_N$, $k_i \in \mathbb{Z}^+, 1, \ldots, N$ are the monomials, and $Z_k \in \mathbb{R}^{n \times n}, \forall k \in \mathcal{K}(g)$ are matrix-valued coefficient. $\mathcal{K}(g)$ is the set of N-tuples obtained as all possible combinations of non-negative integers k_i, $i = 1, \ldots, N$, such that $k_1 + k_2 + \ldots + k_N = g$. To illustrate this notation, consider a homogeneous polynomial of a degree $g = 2$ with $N = 2$. The set $\mathcal{K}$ is given by $\mathcal{K} = \{02, 11, 20\}$ corresponding to the generic form $Z_2(\alpha) = \alpha_2^2 Z_{02} + \alpha_1\alpha_2 Z_{11} + \alpha_1^2 Z_{20}$. This choice for the decision variables provides less conservative results with the increase of g at a price of greater complexity and computational effort. Robust LMIs with parameters in the unit simplex can be fully characterized by means of homogeneous polynomial solutions, without loss of generality [37]. That is, if a solution of a degree g exists, a sequence of LMIs providing sufficient conditions for the existence of homogeneous polynomials of increasing degree $g > g$ can be used, with convergence assured for a large enough g [36]. The LMI conditions, expressed only in terms of the vertices of the system, were obtained with the ROLMIP (Robust LMI Parser) toolbox available at [38]. The toolbox is developed for Matlab and works jointly with YALMIP, returning the entire set of LMIs through simple commands that describe the structure of the matrices involved and the robust LMI conditions to be programmed.

Illustrative Examples

The objective of the examples is to demonstrate the effectiveness and applicability of the proposed approach, and compare the conditions proposed in this paper with other methods from the literature. The routines were implemented in Matlab, version 7.10.0.246 using the programs YALMIP [39] and SeDuMi [40].

Example 1: Consider the discrete-time system [26] given by

$$A = \begin{bmatrix} 0.1 & -0.5 \\ 1 & 0.3+\rho \end{bmatrix}, A_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} -100 & 10 \end{bmatrix}, C_{dy} = \begin{bmatrix} 0 & 0 \end{bmatrix}, D = 0.1, \beta = 0.9$$

$$C_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.5 \end{bmatrix}, D_0 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

where ρ stands for the uncertain parameter taking value in the interval (-0.2, 0.2).

Solving Problem 1 in [26] by using the function *mincx* in the MATLAB LMI Toolbox, we get the minimal value of $\gamma = 5.6802$, whereas the minimal value mentioned in [26] is $\gamma = 2.1217$.

Increasing the bound of ρ to widen the range of uncertainty, in [26], the example is still feasible while for $|\rho| < 0.66$, beyond this interval, the result become infeasible. Applying Theorem 3 and using the proposed Algorithm, for $d_m = 1$ and $d_M = 3$, our result is still feasible for $|\rho| \le 0.8$, and the system is exponentially mean square stable with H_∞ norm bound $\gamma_{opt} = 3.8484$.

Considering initial conditions $\varphi_k = [1 \quad 0]$, the following signal as noise input $\omega_k = 0.1\cos(0.1\pi k)\exp(-0.3k)$ and a randomly generated time-delay $d_k \in [1, 3]$, a time simulation is performed to compute the error signal when the robust filter is obtained through Theorem 3.

$$A_f = \begin{bmatrix} -0.4430 & -0.3623 \\ 0.5120 & 0.3986 \end{bmatrix}, \quad B_f = \begin{bmatrix} 0.0052 \\ 0.0128 \end{bmatrix}, \quad C_f = 10^{-8}\begin{bmatrix} -0.0217 & -0.0033 \\ -0.1136 & -0.0789 \end{bmatrix}, \quad D_f = 10^{-11}\begin{bmatrix} -0.6976 \\ 0.7084 \end{bmatrix} \tag{52}$$

The response of $\tilde{z}_k$ for the system with filter parameter (52) is shown in Fig. 1. From the simulation, it can be noted that the proposed design method achieves desired system performance. We can conclude that our method proves its superiority over the delay-independent one.

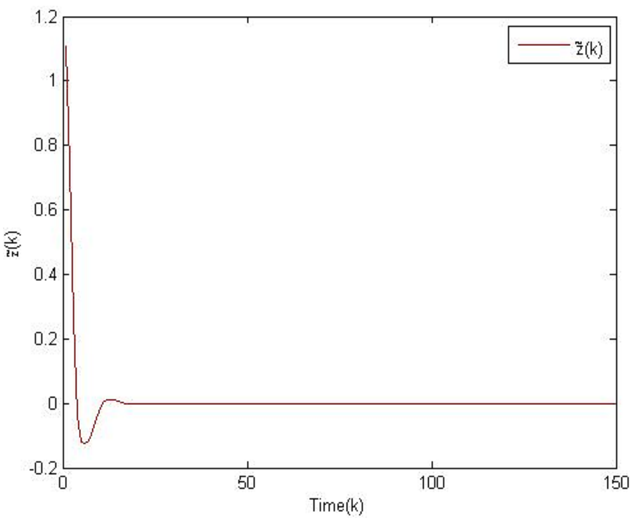

Figure 1. Trajectory of $\tilde{z}_k$.

Example 2: Consider the discrete-time system [10] given by

$$A = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.7+\phi \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & \rho \\ -0.1 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = [1 \quad 1], C_{dy} = [0.2 \quad 0.5], D = 1,$$
$$C_0 = [1 \quad 2], C_d = [0.5 \quad 0.6], D_0 = -0.5$$

where $|\phi| \le 0.2$ and $|\rho| \le \mu$. Table 1 shows the H_∞ performance bounds obtained for $d_m = 1$, $\beta = 1$ and $d_M = 5$ and different values of μ. As can be seen, the proposed approach provides less conservative results than the ones in [10], [33], [34] and [35].

Table1. H_∞ performance bounds for Example 1 using Theorem 3 for $d_m = 1$and $d_M = 5$ and different values of μ.

μ	0.5	0.6	0.7
[34]	2.3662	2.9494	4.4920
[33](g=1)	2.3691	3.0628	4.9838
[33](g=2)	2.3179	2.8175	3.9136
[35](g=1)	2.3179	2.7919	3.6249
[35](g=2)	2.3179	2.7919	3.6171
[10](g=1)	2.2652	2.7215	3.5706
[10](g=2)	2.2651	2.7199	3.5626
Theorem 3 (g=1)	2.0008	2.0032	2.0040
Theorem 3 (g=2)	2.0003	2.0003	2.0004

Considering the augmented system (11) and initial conditions $\varphi_k = [1 \quad -1]$, , the following signal as noise input $\omega_k = 0.1\sin(0.1\pi k)\exp(-0.3k)$ and a randomly generated time-delay $d_k \in [1, 5]$, a time simulation is performed to compute the error signal when the robust filter matrices obtained through Theorem 3 with $d_m = 1$, $\beta = 1$, $d_M = 5$ and g=1 are given by $A_f = \begin{bmatrix} 0.0050 & -0.0263 \\ 0 & -0.5207 \end{bmatrix}$, $B_f = \begin{bmatrix} -0.0226 \\ -1.3068 \end{bmatrix}$, $C_f = [0 \quad -0.5194]$, $D_f = 1.2037$. The responses of z_k and $\hat{z}_k$, for the system with above filter parameter matrices are shown in Fig. 2. From the simulation, it can be concluded that the proposed design method achieves desired system performance for the system subject to uncertainties.

In addition, to see how the missing measurements affect the H_∞ performance of the filtering process, we give the interplay between the missing probability and the optimal H_∞ performance. The relationship of β versus γ is shown in Fig. 3. The results indicate that the bigger the missing probability, the poorer the H_∞ performance, which is reasonable.

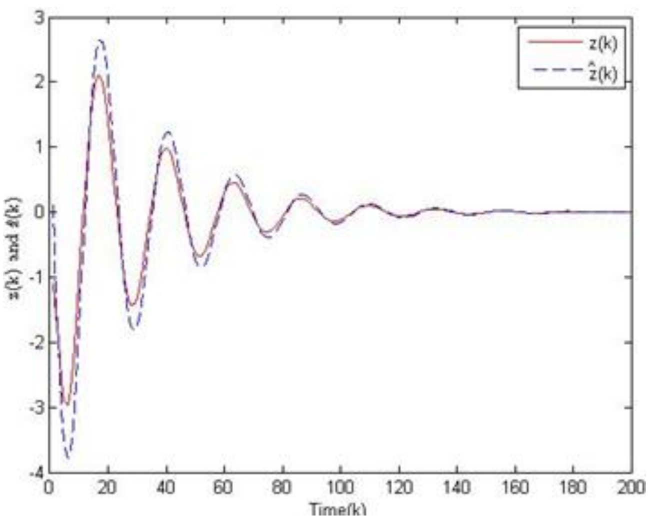

Figure 2. Trajectories of z_k and $\hat{z}_k$.

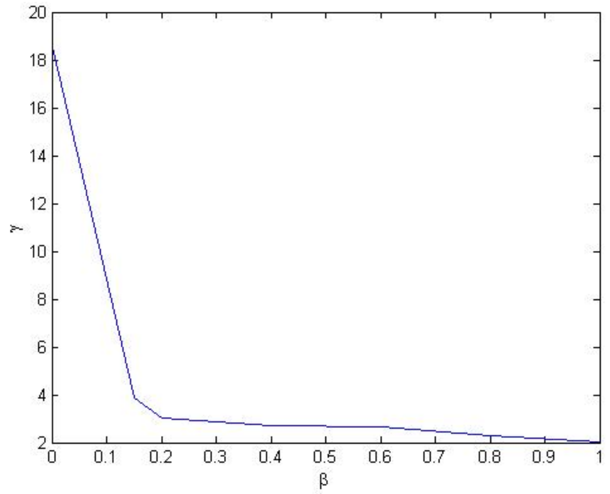

Figure 3. Variation of γ in terms of β.

Conclusion

This paper presents new delay-dependent robust LMI conditions for the design of a robust H_∞ filter for discrete-time uncertain linear systems with missing measurements. The design is based on using the Lyapunov matrices and the slack matrices as polynomially dependent parameters of an arbitrary degree. As this degree increases, the less conservativeness of the results increase; the results are established in terms of LMIs with some tuning parameters. A tuning procedure is proposed to search for the best value of these parameters. Numerical computations are provided to show the improvement of the proposed approach with respect to some existing methods.

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