



Hierarchical Coordination for Multi-Robot Systems with Region-Based Tracking Control

Chao-Wei Lin, Luis A. Sanchez-Porras, and Yen-Chen Liu*

Department of Mechanical Engineering, National Cheng Kung University, Tainan 70101, Taiwan

(Received 31 July 2014; Accepted 27 November 2014; Published on line 1 March 2015)

* Corresponding author email: yliu@mail.ncku.edu.tw

DOI: [10.5875/ausmt.v5i1.839](https://doi.org/10.5875/ausmt.v5i1.839)

Abstract: This paper presents a control framework for hierarchical coordination of multi-robot systems using leader-follower and region-based approaches. The leader robot is controlled to converge to and circulate along a closed curve that is defined by an implicit function. In the proposed control framework, only the leader robot has the knowledge of the desired curve, and the follower robots are controlled to track the curve indirectly through the leader robot. To enhance redundancy and flexibility of multi-robot systems, in practice the follower robots are controlled to stay freely within a designated region surrounding the leader robot. Moreover, the group of follower robots is able to circulate in the specified region to enlarge the sensing areas. The stability and convergence of the proposed multi-robot system with the addressed control algorithms are studied. Simulation and experimental results on mobile robots are presented to validate the performance of the hierarchical control framework for multi-robot systems.

Keywords: Multi-robot system, hierarchical coordination, leader-follower approach, region-based control

Introduction

The study of multi-robot systems has attracted significant attention from researchers in the robotics community because they provide higher redundancy and capability of handling complex tasks in various applications. Such robotic systems are also superior to single robots for enhancing robustness against failures and increasing flexibility to system and environmental changes. A multi-robot system is composed of a group of moving robots whose capabilities as a whole are better suited to solving certain missions than a single robot [25]. The cooperation between multiple robots facilitates negotiating complicated tasks even if one of the robots fails [8],[10]. The study of multi-robot systems combines control theory, network communication, and sensory technology in a field with great potential. Many applications, for example formation control [1],[9], environmental monitoring [10][30], and cooperation [23][27] can benefit from using such multiple robot frameworks.

In studies of multi-robot systems, the most popular

control approaches are virtual structure [3][16][31], behavior-based [2][4], motion planning [28][29], and leader-follower approaches [6][7],[17]. A virtual structure approach generates reference trajectories for all robots with respect to a common reference, called the virtual center, in order to achieve a geometric formation. However, relying too heavily on mutual communication between robots and maintaining a rigid formation limits the flexibility of such frameworks in real applications. To combine different behaviors in a multi-robot system, several different types of motions are weighted to generate an ultimate behavior for the robots utilizing a behavior-based approach. Although various control inputs can be taken into account simultaneously, stability of the robotic control system using this approach is difficult to study.

Leader-follower and region-based approaches are also popular in the development of multi-robot systems. In a leader-follower framework [6][7][17] one or more robots are identified as leader(s) which have knowledge of the desired global missions, while the others are identified as followers which maintain specific distances and orientations with respect to the leader(s). Although



this approach can control a group of robots to maintain a desired formation, there is limited flexibility for changing the formation. Region-based control [5][12][24] can provide flexibility for multi-robot systems to move around and stay within a predefined region. However, all robots in such a system must know the desired trajectory and regions; hence, the communication load depends significantly on the number of robots in the multi-agent system.

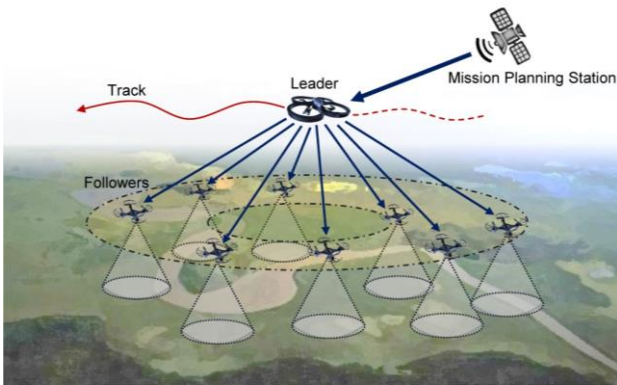


Figure 1. An illustrated example of region-based curve tracking for a multi-robot system in environmental monitoring.

This paper proposes a hierarchical coordination control framework for multi-robot systems to monitor a specified area with reduced communication and higher flexibility. To avoid the disadvantages of the aforementioned approach for multi-robot systems, the control architecture is developed under the leader-follower approach. The proposed control algorithms can be used to coordinate a group of mobile robots hierarchically in environmental monitoring, as shown in Fig. 1. The leader robot is designed to receive from the mission planning station a global command, which is related to the task of the group as a whole. While executing the global task, the information of the leader robot is transmitted to the follower robots to accomplish local tasks. The advantages of this framework are that the

Chao-Wei Lin received the B.S. degree from National Cheng Kung University, Tainan, Taiwan, 2013 and currently works toward his master degree at the same institute. His research interests include bilateral teleoperation and distributed multi-robot systems.

Luis A. Sanchez-Porras received the B.S. degree from Kun Shan University, Tainan, Taiwan, and M.S. degree from National Cheng Kung University, Tainan, Taiwan. Since November 2014, he has been working at Balance Ambiental S.A. in Costa Rica, as a Quality Control Engineer. His current research interests include mobile robots, automation, and renewable energies.

Yen-Chen Liu received the B.S. and M.S. degrees from National Chiao Tung University, Hsinchu, Taiwan, in 2003 and 2005, respectively, and the Ph.D. degree from the University of Maryland, College Park, MD, USA, in 2012, all in mechanical engineering. Since August 2012, he has been with the Department of Mechanical Engineering, National Cheng Kung University, Tainan, Taiwan, currently as an Assistant Professor. His current research interests include control of robotic system, bilateral teleoperation, multi-robot system, semi-autonomous system, and human-robot interaction.

follower robots only need limited sensing and communication capabilities and the group of multi-robot system can still achieve the global goal through the leader robot.

The movement of the leader robot in the proposed control framework is accomplished through curve tracking control, a well-known research area with many applications in perimeter surveillance, environmental protection, and cooperative searching [15][23]. By using a vector field-based technique, the curve tracking problem can be specified with the design of potential functions. The gradient of potential functions is used in control laws and considered in a Lyapunov function to demonstrate the stability of a mobile robot system [21]. Moreover, vector fields may also be locally modified for collision avoidance and circulation terms without compromising the global properties of the system [22]. A previous work on a curve tracking problem [15] used a navigation function to control a group of robots to circulate the boundary of star shape maintaining a non-zero velocity. Addressing the same problem, [11] defined a time-varying curve in n -dimensions and used a vector field to compensate for the time-varying nature of the curve. Recently, [23] introduced a decentralized controller to make a group of robots circulate a curve that can be implemented in an end-of-line industrial scenario.

To increase flexibility, the follower robots are controlled to move within a given region surrounding the leader robot. Therefore, the idea of region-based control for a multi-robot system is adopted in this paper. The control algorithm for a group of robots to converge to a time-varying region with different shapes was addressed in [5]. Such shapes are constructed by a region-objective function defined by artificial potential functions. Following the steps in the previous research, [13] presented a region-based control related to the union of various basic shapes. To generate complex shapes, [12] introduced the concept of decomposing swarm robots into multiple sub-groups. Recently, the region-based control for swarm robots was applied to bilateral human-swarm interaction [18]. Although the group of mobile robots can converge to a specified region by using the aforementioned control algorithms, these robots are not able to circulate within the given regions. Therefore, the sensing or monitoring area can only be enlarged by adding more robots.

In this paper, a leader-follower approach is adopted for a multi-robot system to track a given curve by circulating within a desired region. The control system for the leader robot is adopted from the results in [23], but in the proposed hierarchical coordination framework only the leader robot knows the curve function. The follower robots, without the information of the desired curve, can

converge to a region surrounding the leader robot and circulate to enlarge the monitoring area. Under this control framework, only the leader robot requires powerful communication capability, and the follower robots can focus on sensing and monitoring the designated environment by tracking the curve indirectly through the leader robot.

The rest of this paper is organized as follows. The model of the mobile robots and problem formulation are described in Section 2. In Section 3, we present the controller algorithms for the leader and follower robots with stability and convergence analysis of the multi-robot system. Simulation and experimental results are subsequently addressed in Section 4, and conclusions are presented in Section 5.

Problem Formulation

In the proposed multi-robot system, we consider a group of $N+1$ mobile robots, which are modeled by holonomic dynamics given as [14][23][24]

$$\begin{cases} \dot{q}_i = v_i \\ \dot{v}_i = u_i \end{cases} \quad (1)$$

where $q_i = [x_i, y_i]^T \in W$ denotes the position vector of the i^{th} robot, $v_i \in R^2$ is the velocity of the i^{th} robot, and $u_i \in R^2$ is the control input that will be designed subsequently. The group of mobile robots moves within the workspace $W \subset R^2$. For the mobile robots with non-holonomic constraint, we assume that the individual dynamics can be transformed into (1) by some diffeomorphic state transformation.

The proposed hierarchical coordination control framework is composed of a leader robot and a group of N follower robots. The leader robot is denoted by q_ℓ , and the follower robots are denoted by $q_i, i = 1, \dots, N$. By following the aforementioned notation, the configuration of the multi-robot system is defined as $q = [q_\ell^T, q_1^T, \dots, q_i^T, \dots, q_N^T]^T \in Q$, where $Q \in R^{2 \times (N+1)}$ is the configuration space of the multi-robot system. In the sequel, we denote $\mathbf{q}_\ell = [q_\ell^T, v_\ell^T]^T$ as the state of the leader robot and $\mathbf{q}_f = [\mathbf{q}_1^T, \dots, \mathbf{q}_i^T, \dots, \mathbf{q}_N^T]^T$ with $\mathbf{q}_i = [q_i^T, v_i^T]^T$ as the state of the follower robots.

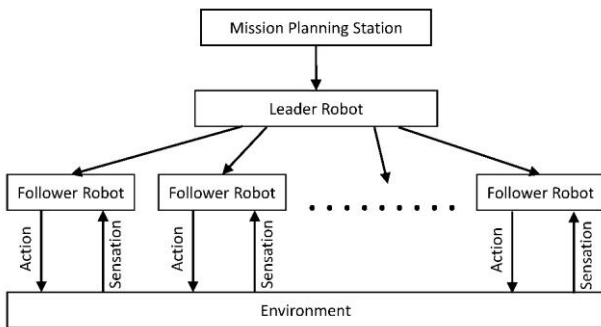


Figure 2. The hierarchical coordination framework for the proposed multi-robot system.

The objective of this paper is to design a hierarchical coordination framework for multi-robot systems to monitor, detect, or examine a designated area with fewer robots, reduced communication, and higher flexibility. The proposed system is composed of a leader robot and a group of follower robots. The leader robot communicates to a mission planning station, which can provide the leader robot with information needed to track and orbit the desired curve. The group of follower robots, which has no information about the desired trajectory, can only communicate with the leader robot. Based on the information received from the leader robot, the follower robots are controlled to approach a prescribed region surrounding the leader robot and circulate within the region to execute the assigned mission such as the surveillance of an environment.

In the proposed control framework, the leader robot tracks a curve C , which is defined by an implicit function $f_c(x, y) : R^2 \rightarrow R$. In general, we assume that the desired curve C defined by the implicit function $f_c(x, y)$ is a closed, simple, and smooth planar track without self-intersection [14]. The hierarchical framework for the proposed multi-robot system is illustrated in Fig. 2. Under the assumption that the leader robot has long-distance communication capability, the information of the curve C , which is generated from a control station, is available only to the leader robot. The goal of the leader robot is to move towards the curve C and circulate along it. In the multi-robot system, the group of follower robots is assumed to be embedded with limited communication capability so that the follower robots can only receive signals from the leader robot. The goal of the follower robots is to converge to a region surrounding the leader robot. Using embedded powerful sensor devices, the follower robots can orbit the leader robots and thus sense, monitor, and/or observe the designated environment simultaneously.

Based on the formulated system, we summarize several assumptions that will be considered in this paper.

Assumption 1 The curve $C \in R^2$ defined by a time-invariant implicit function $f_c(x, y) = 0$ is only available to the leader robot.

Assumption 2 The individual follower robots can always maintain connection to the leader robot, and there is no communication between any of the follower robots.

Assumption 3 All robots have mounted range sensors that can detect the relative distance to other robots within a prescribed range R_d . The sensing range is bidirectional.

Assumption 4 The velocity of the mobile robots is bounded, and v_ℓ of the leader robot is smaller than the velocities v_i of the follower robots.

Throughout this paper, we use the following notations: $R := (-\infty, \infty)$, $R^+ := (0, \infty)$, and $\| \cdot \|$ stands for the Euclidean norm. Moreover, the notations $\nabla_\ell = \partial / \partial q_\ell$

and $\nabla_i = \partial / \partial q_i$ are respectively used in this paper to obtain the gradient of a function with respect to q_ℓ and q_i . The following section discusses the methodology used to construct the controller for q_ℓ and q_i .

Controller Design and System Analysis

Controller Design

The leader robot tracks and circulates along a curve assigned by the mission planning station. The control input to the leader robot is adopted from [23] and is given as

$$u_\ell = \underbrace{-k_\ell \nabla_\ell f_1^2(q_\ell)}_{u_\ell^t} - \underbrace{b_\ell v_\ell}_{u_\ell^d} + \underbrace{u_\ell^c}_{u_\ell^c} \quad (2)$$

where u_ℓ^t represents the curve tracking control, u_ℓ^d is the damping term, and u_ℓ^c denotes the circulating control. In (2), $b_\ell \in R^+$ is a damping coefficient, and $k_\ell \in R^+$ is a positive control gain. The tracking term u_ℓ^t is used to guarantee that the leader robot can approach the desired curve C . As mentioned in Section 2, $f_1^2(q_\ell)$, used to define the curve C , is continuously differentiable and equal to zero when q_ℓ is on the curve C . The control term $u_\ell^t = -k_\ell \nabla_\ell f_1^2(q_\ell)$ generates a vector field by the gradient descent of $f_1^2(q_\ell)$ to force the leader robot to converge to the desired curve C . Even though the implicit function gives the information about the curve C , we assume that only the leader robot is able to access f_1 .

The control term u_ℓ^c in (2), used by the leader robot to circulate around the curve C , is defined by [23]

$$u_\ell^c = \begin{cases} \omega_\ell \cdot R_\theta(q_\ell) \cdot e'_\ell(q_\ell) & \text{if } f_1(q_\ell) \neq 0 \\ \omega_\ell \cdot R_\theta(q_p) \cdot e'_\ell(q_p) & \text{otherwise} \end{cases} \quad (3)$$

where $\omega_\ell \in R$ is a constant value for the circulating velocity of the leader robot, $e'_\ell(q_\ell) := \nabla_\ell f_1^2(q_\ell) / \|\nabla_\ell f_1^2(q_\ell)\|$ denotes a unit vector pointing towards the direction of the decreasing $f_1^2(q_\ell)$, and q_p represents any point perpendicular to the position of the leader robot q_ℓ with respect to the tangent of the curve C . The matrix $R_\theta(\cdot) \in R^{2 \times 2}$ in (3) is a rotational matrix that is defined by [23]

$$R_\theta(q_\ell) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}, \quad (4)$$

where

$$\theta(q_\ell) = \begin{cases} -\pi/2 & \text{if } f_1(q_\ell) > 0 \\ \pi/2 & \text{otherwise} \end{cases} \quad (5)$$

The transformation matrix $R_\theta(\cdot)$ is used to change the direction of the configuration vector of the mobile robot. Based on (3) with (4) and (5), the leader robot will circulate clockwise if $\omega_\ell > 0$, and counterclockwise if $\omega_\ell < 0$.

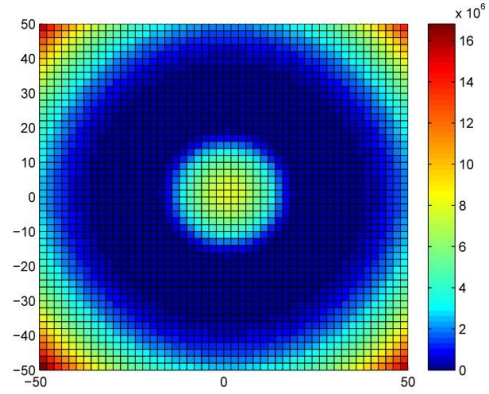


Figure 3. The potential function, $f_1(q_{i\ell}) + f_2(q_{i\ell})$, used to define a region for the follower robots to circulate the leader robot, which is located at $[0; 0]$

By denoting $q_{i\ell} := q_i - q_\ell$ and $v_{i\ell} := v_i - v_\ell$ for $i = 1, \dots, N$ as the position and velocity errors between the leader robot and the i^{th} follower robot, the control input for the follower robots is given as

$$u_i = \underbrace{-k_1 \nabla_i f_1^2(q_{i\ell})}_{u_i^r} - \underbrace{k_2 \nabla_i f_2^2(q_{i\ell})}_{u_i^d} - \underbrace{b_{fi} v_{i\ell}}_{u_i^c} + u_i^a + u_i^c, \quad (6)$$

where u_i^r represents the region-based control, u_i^d is the relative damping term between the i^{th} follower robot and the leader robot, u_i^a is the control term for collision avoidance, and u_i^c denotes the circulating control input. In (6), $k_1, k_2 \in R^+$ are control gains for region-based control, $b_{fi} \in R^+$ is the damping coefficient for the i^{th} follower robot, and $f_1^2(\cdot), f_2^2(\cdot)$ are potential functions used to define the region for the follower robots. The control term u_i^r guarantees that the follower robots will stay within a given region, which is assumed to be connected and convex. For simplicity, we consider a ring-shaped area as the desired region in the proposed control system for the multi-robot system. Thus, the potential functions $f_1(\cdot), f_2(\cdot) \in R^2 \rightarrow R$ are defined by

$$f_1(q_{i\ell}) = \begin{cases} \|q_{i\ell}\|^2 - r_1^2 & \text{if } \|q_{i\ell}\| > r_1 \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

and

$$f_2(q_{i\ell}) = \begin{cases} r_2^2 - \|q_{i\ell}\|^2 & \text{if } \|q_{i\ell}\| < r_2 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where $r_1, r_2 \in R^+$ are the constant radii of two circles such that $r_1 > r_2$. As seen in Fig. 3, by using the control term u_i^r ,

the vector field forces the follower robots to move into and stay within the donut-shaped region.

Remark 1 The desired region defined in (7) and (8) for the follower robots need not be circular, and various shapes can be considered to establish the desired region [5],[13]

The collision avoidance term u_i^a is used for follower robots to keep a distance from the other follower robots while moving within the prescribed region. By denoting $q_{ij} := q_i - q_j$, the distance between the i^{th} and the j^{th} follower robots is defined by $\|q_{ij}\|$. Hence, the control term for collision avoidance is given as

$$u_i^a = -k_a \sum_{j=1, j \neq i}^N \nabla_i f_{ij}^a(q_{ij}), \quad (9)$$

where $k_a \in R^+$ is a control gain, and f_{ij}^a is the avoidance function given as [19]

$$f_{ij}^a(q_{ij}) = \sum_{i=1}^n \left(\min \left\{ 0, \frac{\|q_{ij}\|^2 - R_d^2}{\|q_{ij}\|^2 - r_d^2} \right\} \right)^2, \quad (10)$$

where $R_d \in R^+$ is the sensing distance as mentioned in Assumption 3, and $r_d \in R^+$ denotes the safe distance which is the smallest distance between any two follower robots. The gradient descent term for collision avoidance in (9) is then given as

$$\nabla_i f_{ij}^a(q_{ij}) = \begin{cases} 0, & \|q_{ij}\| \geq R_d \\ 4 \left[\frac{(R_d^2 - r_d^2)(\|q_{ij}\|^2 - R_d^2)}{(\|q_{ij}\|^2 - r_d^2)^3} \right] q_{ij}^T, & R_d > \|q_{ij}\| > r_d \\ \text{not defined}, & \|q_{ij}\| = r_d \\ 0, & \|q_{ij}\| < r_d \end{cases}$$

Under Assumption 3, we define the sensing region by the pairwise regions

$$S_{ij} = \{q_i \mid q_i \in W, \|q_{ij}\| \leq R_d\}. \quad (11)$$

Furthermore, the avoidance set for each pair of the follower robots is given as

$$\Omega_{ij} = \{q_i \mid q_i \in W, \|q_{ij}\| \leq r_d\}. \quad (12)$$

Hence, the overall sensing and avoidance regions are given as $S = \cup_{i,j} S_{ij}$ and $\Omega = \cup_{i,j} \Omega_{ij}$ for $i = 1, \dots, N, j \neq i$, respectively.

The potential function of (10) guarantees that the distance between any pair of follower robots will not smaller than the safe distance r_d . Therefore, the collision avoidance in the mobile robot network can be ensured such that $q_i \in W / \Omega, i = 1, \dots, N$. In practice, the prescribed ranges R_d and r_d can be determined based on the sensory devices installed on the follower robots.

The last control term to be mentioned for the follower robots is u_i^c . The purpose of u_i^c is to drive the follower robots to circulate within the region defined by (7) and (8). The control term u_i^c is given as

$$u_i^c = \omega_{fi} \cdot R_\theta(\theta) \cdot e_i^c, \quad (13)$$

where $\omega_{fi} \in R$ is a constant value for the circulating

velocity of the follower robots, $R_\theta(\theta)$ is given as (4), and $e_i^c \in R^2$ is a unit vector given by $e_i^c := -q_{i\ell} / \|q_{i\ell}\|$. Different from the leader robot which has the desired curve to track, the follower robots can only circulate along the leader robot so that e_i^c is used to define the circulating direction. Therefore, the follower robots can track the desired curve C indirectly via the leader robot based only on their knowledge of the leader robot's position and velocity without receiving the information of the implicit function $f_i(x, y)$, which is used to define the curve C .

System Analysis

The basic idea behind this research is to make a group of follower robots converge to a region orbiting the leader robot. The follower robots do not recognize the curve, and can only track the curve indirectly by following the leader robot that has the knowledge of the desired curve. The behaviors of the proposed multi-robot system are studied in this section.

We first address the convergence of the leader. By substituting the control law (2) into the dynamics model (1), the closed-loop control system is given as

$$\begin{cases} \dot{q}_\ell = v_\ell \\ \dot{v}_\ell = -k_\ell \nabla_\ell f_\ell^2(q_\ell) - b_\ell v_\ell + u_\ell^c \end{cases} \quad (14)$$

Since the leader robot is controlled to approach towards the curve C and circulates along it, the entire motion can be divided into two orthogonal motions which are parallel and perpendicular to the curve C . In the following analysis, we denote $q_{\ell\parallel}^c$ and $q_{\ell\perp}^c$ respectively for signals of the leader robot parallel and perpendicular to C . By following Assumption 1, we first state a definition for curve tracking of the leader robot.

Definition 1 The leader robot q_ℓ is said to converge to a curve C that is defined by $f_i: R^2 \rightarrow R$ if $\nabla_\ell f_i^2(q_\ell) = 0$ for future time.

This definition implies that the function $f_i^2(q_\ell)$ has reached a minima, and this minima value can be only a point on the curve C . Therefore, the control term u_ℓ^c will be zero as long as q_ℓ stays within the curve C . It is noted that this condition does not imply that the robot can circulate along the curve because u_ℓ^c always acts perpendicular to the curve. Since the motion of the leader robot is not influenced by the follower robots, the analysis can be described individually and the proof of the next proposition is adopted from [23].

Proposition 1 Consider the leader robot in a multi-robot system with the dynamics model (1) and the control law (2). The leader robot asymptotically converges to the

curve C defined by $f_t(x, y)$ and never leaves the curve.

Proof

Since this proposition only considers the curve tracking of the leader, (14) can be simplified as

$$\begin{cases} \dot{q}_{t\perp}^c = v_{t\perp}^c \\ \dot{v}_{t\perp}^c = -k_f \nabla_{\ell} f_t^2(q_t) - b_t v_{t\perp}^c \end{cases} \quad (15)$$

Where u_{ℓ}^c is removed because it only contributes to the motion of leader robot parallel to the curve. Moreover, the control term $-k_f \nabla_{\ell} f_t^2(q_t)$ has no notation \perp since this term only influences the motion perpendicular to the curve C .

Consider the Lyapunov function candidate for the motion of the leader robot perpendicular to the curve C as

$$V_{t\perp}(\mathbf{q}_t) = k_t f_t^2(q_t) + \frac{1}{2} v_{t\perp}^{cT} v_{t\perp}^c. \quad (16)$$

Taking the time-derivative of $V_{t\perp}$ along the trajectory of the perpendicular motion of the leader robot (15), we obtain

$$\begin{aligned} \dot{V}_{t\perp}(\mathbf{q}_t) &= v_{t\perp}^{cT} k_t \nabla_{\ell} f_t^2(q_t) + v_{t\perp}^{cT} \dot{v}_{t\perp}^c \\ &= v_{t\perp}^{cT} (k_t \nabla_{\ell} f_t^2(q_t) - k_t \nabla_{\ell} f_t^2(q_t) - b_t v_{t\perp}^c) \\ &= -b_t v_{t\perp}^{cT} v_{t\perp}^c \leq 0. \end{aligned} \quad (17)$$

By invoking LaSalle's invariance principle [26], the system solution approaches the largest invariant set $v_{t\perp}^c \equiv 0$. Therefore, we conclude that $\nabla_{\ell} f_t^2(q_t)$ approaches zero asymptotically from observing (15). According to Definition 1, the leader robot converges to the curve C defined by $f_t(x, y)$. \square

In addition to the perpendicular motion, the leader robot has to move along the given curve C . This behavior is discussed in the next proposition.

Proposition 2 Consider the leader robot in a multi-robot system with the dynamics model (1) and the control law (2). If the leader robot is on the curve C , then the robot moves along the curve at a constant speed ω_{ℓ} / b_{ℓ} .

Proof

The motion of the closed-loop leader robot control system parallel to the curve C can be given as

$$\begin{cases} \dot{q}_{t\parallel}^c = v_{t\parallel}^c \\ \dot{v}_{t\parallel}^c = -b_{\ell} v_{t\parallel}^c + \omega_{\ell} \end{cases} \quad (18)$$

where the control term u_{ℓ}^c vanishes because it only affects the motion of the leader robot perpendicular to the curve. Since $R_{\theta}(q_t) \cdot e_{\ell}^t(q_t)$ and $R_{\theta}(q_p) \cdot e_{\ell}^t(q_p)$ only represent the direction of the circulating control input, u_{ℓ}^c is equal to ω_{ℓ} in (18). For the second subsystem of (18) with the definition that $\mathbf{e}_{t\parallel} := v_{t\parallel}^c - \omega_{\ell} / b_{\ell}$, we consider the Lyapunov function candidate

$$V_{t\parallel}(\mathbf{e}_{t\parallel}) = \frac{1}{2} \mathbf{e}_{t\parallel}^T \mathbf{e}_{t\parallel} = \frac{1}{2} (v_{t\parallel}^c - \omega_{\ell} / b_{\ell})^T (v_{t\parallel}^c - \omega_{\ell} / b_{\ell}).$$

The time-derivative of the above function is

$$\begin{aligned} \dot{V}_{t\parallel}(\mathbf{e}_{t\parallel}) &= (v_{t\parallel}^c - \omega_{\ell} / b_{\ell})^T \dot{v}_{t\parallel}^c \\ &= (v_{t\parallel}^c - \omega_{\ell} / b_{\ell})^T (-b_{\ell} v_{t\parallel}^c + \omega_{\ell}) \\ &= -b_{\ell} (v_{t\parallel}^c - \omega_{\ell} / b_{\ell})^T (v_{t\parallel}^c - \omega_{\ell} / b_{\ell}) \\ &= -b_{\ell} \mathbf{e}_{t\parallel}^T \mathbf{e}_{t\parallel} < 0. \end{aligned} \quad (19)$$

Thus, we conclude that $\mathbf{e}_{t\parallel}$ goes to zero asymptotically. Therefore, $\lim_{t \rightarrow \infty} v_{t\parallel}^c = \omega_{\ell} / b_{\ell}$. \square

We next present the system behavior of the follower robots. The objective of the group of follower robots is to stay in a designated region moving with the leader robot, and circulating within the given area. By substituting the controller (6) into the dynamics model (1), the closed-loop control system of the follower robots is given as

$$\begin{cases} \dot{q}_i = v_i \\ \dot{v}_i = -k_1 \nabla_i f_1^2(q_{i\ell}) - k_2 \nabla_i f_2^2(q_{i\ell}) - b_{\beta} v_i + u_i^a + u_i^c \end{cases} \quad (20)$$

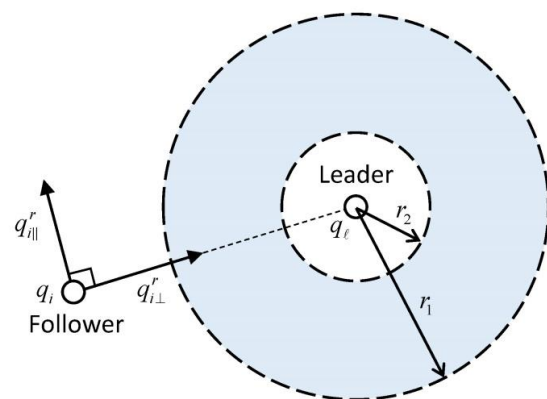


Figure 4. An illustration of the notation for the movement of the follower robots.

Similar to the analysis for the leader robot, the behaviors of the follower robots can also be divided into two orthogonal motions. Since the follower robots are moving towards a region with the leader robot as the center, we denote these two motions as $q_{i\perp}^r$ and $q_{i\parallel}^r$, as

shown in Fig. 4.

For the follower robots, the control term u_i^r separates the entire workspace into three sub-regions that $\|q_{i\ell}\| > r_1$, $r_1 \geq \|q_{i\ell}\| \geq r_2$, and $\|q_{i\ell}\| < r_2$. From Fig. 4, the vector field forces the follower robots moving towards the region of $r_1 \geq \|q_{i\ell}\| \geq r_2$ that leads to $u_i^r = 0$. For the region-based tracking control, we consider only the motion of the follower robot perpendicular to the region defined by $f_1(q_{i\ell})$ and $f_2(q_{i\ell})$. The group of follower robots is said to converge to a designated region if the following definition is satisfied.

Definition 2 The follower robots q_i are said to converge to a region that is defined by $f_1(q_{i\ell})$ and $f_2(q_{i\ell})$ if the control term $u_i^r = 0$.

In the next proposition, we study the convergence of the follower robots to the desired region. We first investigate the region-based control for the mobile robot network when the leader robot is stationary.

Proposition 3 Consider a group of N follower robots given by (1) in a multi-robot system with the controller (6). If the leader robot is stationary, then the follower robots converge to the desired region defined by (7) and (8), and the set Ω is avoidable.

Proof

Since this proposition considers the convergence of the follower robots to the given region, the closed-loop dynamics can focus on the motion of the follower robots heading towards the leader robot (the center of the designated region). Thus, the closed-loop control system is given as

$$\begin{cases} \dot{q}_{i\perp}^r = v_{i\perp}^r \\ \dot{v}_{i\perp}^r = -k_1 \nabla_i f_1^2(q_{i\ell}) - k_2 \nabla_i f_2^2(q_{i\ell}) - b_{\bar{f}_i} v_{i\perp}^r + u_{i\perp}^{ar} \end{cases} \quad (21)$$

where the control term u_i^c is not included because u_i^c only contributes to the rotational motion of the follower robots.

Consider the Lyapunov function candidate for the group of follower robots as

$$\begin{aligned} V_{f\perp}(\mathbf{q}_f) &= \sum_{i=1}^N (k_1 f_1^2(q_{i\ell}) + k_2 f_2^2(q_{i\ell}) \\ &+ \frac{1}{2} v_{i\perp}^{rT} v_{i\perp}^r + k_a \sum_{j>i} f_a^2(q_{ij})). \end{aligned} \quad (22)$$

By taking the time-derivative of the above function with the substitution of (21) and $v_\ell = 0$, we have

$$\begin{aligned} \dot{V}_{f\perp}(\mathbf{q}_f) &= \sum_{i=1}^N (k_1 \nabla_i f_1^2(q_{i\ell}) v_i + k_2 \nabla_i f_2^2(q_{i\ell}) v_i + v_{i\perp}^{rT} \dot{v}_{i\perp}^r) \\ &+ k_a \sum_{i=1}^N \sum_{j>i} (\nabla_i f_a^2(q_{ij}) v_i + \nabla_j f_a^2(q_{ij}) v_j) \\ &= -\sum_{i=1}^N b_{\bar{f}_i} v_{i\perp}^{rT} v_{i\perp}^r - k_a \sum_{i=1}^N \sum_{j>i} \nabla_i f_a^2(q_{ij}) v_i \\ &+ k_a \sum_{i=1}^N \sum_{j>i} (\nabla_i f_a^2(q_{ij}) v_i + \nabla_j f_a^2(q_{ij}) v_j). \end{aligned} \quad (23)$$

Since the distance sensing is bidirectional (Assumption 3) and the collision avoidance function is undirected, we have $\nabla_i f_a^2(q_{ij}) = -\nabla_j f_a^2(q_{ij})$. Therefore,

$$\begin{aligned} &-k_a \sum_{i=1}^N \sum_{j>i} \nabla_i f_a^2(q_{ij}) v_i + k_a \sum_{i=1}^N \sum_{j>i} (\nabla_i f_a^2(q_{ij}) v_i \\ &+ \nabla_j f_a^2(q_{ij}) v_j) = 0. \end{aligned}$$

Consequently, the function $\dot{V}_{f\perp}$ becomes

$$\dot{V}_{f\perp}(\mathbf{q}_f) = -\sum_{i=1}^N b_{\bar{f}_i} v_{i\perp}^{rT} v_{i\perp}^r \leq 0. \quad (24)$$

As $V_{f\perp}(\mathbf{q}_f) \geq 0$ and $\dot{V}_{f\perp}(\mathbf{q}_f) \leq 0$, the function $V_{f\perp}(\mathbf{q}_f)$ exists and is finite as $t \rightarrow \infty$. Hence, the function $f_a(q_{ij})$ is bounded so that the set Ω is avoidable. From the boundedness of the function $V_{f\perp}(\mathbf{q}_f)$, we obtain that $\dot{v}_{i\perp}^r$ is bounded. Integrating (24) from $[0, t]$ with bounded $V_{f\perp}(\mathbf{q}_f)$ results that $v_{i\perp}^r$ is square integrable. By invoking Barbalat's lemma [26], $\lim_{t \rightarrow \infty} v_{i\perp}^r = 0$. From (21), the follower robots avoid colliding with other robots and converge to the designated region. \square

Next, we address the result of the proposed multi-robot system with a moving leader robot. If the leader robot is not stationary, then the follower robots can converge to the designated region only if v_ℓ is relatively smaller than v_i (Assumption 4). The analysis is presented in the following proposition.

Proposition 4 Consider a group of N follower robots given by (1) in a multi-robot system with the controller (6). If the velocity of the leader robot v_ℓ is smaller than the velocities of the follower robots v_i , then the follower robots converge to the desired region defined by (7) and (8), and the set Ω is avoidable.

Proof

By following the proof in Proposition 3, if $v_\ell \neq 0$ for the time-derivative of the (22), then we have

$$\begin{aligned} \dot{V}_{f\perp}(\mathbf{q}_f) &= \sum_{i=1}^N (k_1 \nabla_\ell f_1^2(q_{i\ell}) v_\ell + k_2 \nabla_\ell f_2^2(q_{i\ell}) v_\ell \\ &- b_{\bar{f}_i} v_{i\perp}^{rT} v_{i\perp}^r + b_{\bar{f}_i} v_{i\perp}^{rT} v_\ell). \end{aligned} \quad (25)$$

Based on the definition of the region (7) and (8), we find that $\nabla_\ell f_1^2(q_{i\ell}) = -4 \|q_{i\ell}\| f_1(q_{i\ell}) \leq 0$, $\nabla_\ell f_2^2(q_{i\ell}) = -4 \|q_{i\ell}\| f_2(q_{i\ell}) \leq 0$. By denoting $B_{i1} = 4 \sup\{\|q_{i\ell}\| \cdot \|f_1(q_{i\ell})\|\}$ and $B_{i2} = 4 \sup\{\|q_{i\ell}\| \cdot \|f_2(q_{i\ell})\|\}$ which are dependent on the initial position of the mobile robots in the network, the time-derivative of the function $V_{f\perp}(\mathbf{q}_f)$ becomes

$$\begin{aligned} \dot{V}_{f\perp}(\mathbf{q}_f) &\leq \sum_{i=1}^N (-b_{\bar{f}_i} \|v_{i\perp}^r\|^2 + B_{i1} \|v_\ell\| + B_{i2} \|v_\ell\| + b_{\bar{f}_i} \|v_{i\perp}^r\| \cdot \|v_\ell\|) \\ &= \sum_{i=1}^N \left(-\frac{b_{\bar{f}_i}}{2} \|v_{i\perp}^r\|^2 + \frac{b_{\bar{f}_i}}{2} \|v_\ell\|^2 + B_i \|v_\ell\|\right), \end{aligned}$$

where $B_i := B_{i1} + B_{i2}$. Under Assumption 4 (with large b_ℓ and/or small ω_ℓ), the above function can be considered



as $\dot{V}_{f\perp}(\mathbf{q}_f) \leq 0$. Therefore, this proposition can be complete by following the proof addressed in Proposition 3.

When the group of robots converges to the designated regions, the velocity term v_i has only a component parallel to the curve as $u_i^r = 0$. The circulating motion of the follower robots can be demonstrated by the following proposition.

Proposition 5 Consider a group of N follower robots given by (1) and the controller (6). If the follower robots are inside the designated region, then the follower robots circulate in this region while surrounding the leader robot.

The proof of this proposition can be accomplished by following the proof of Proposition 2 with the analysis of collision avoidance in the proof of Proposition 4.

Remark 2 In the proposed hierarchical multi-robot system, the leader and the follower robots are controlled to circulate after converging to a curve or a region. The centrifugal force resulting from the revolution will influence the movement of the mobile robots. Since there is only a line to track for the leader robot, the presence of centrifugal force is not an issue. However, the centrifugal force will drive the follower robots moving towards the periphery of the outer circle defined by (8). To avoid this situation and enlarge the monitoring area of the follower robots, the control term $\omega_f^2 \|q_{it}\| e_i^c$ can be added to u_i for the follower robots. With the addition of the centrifugal force, the follower robots will enter the desired region and keep away from the periphery of the outer circle. More details will be discussed in the next section via simulation and experiment.

Validation Results

Numerical Examples

The proposed hierarchical coordination framework for the proposed multi-robot control system is validated in this section via numerical simulation. The holonomic dynamics model (1) is considered in the simulation, where the leader robot is considered to track a circular curve with a radius of 200 cm and the center at the origin. Thus, we have the potential function in the control term u_i^l that $f_l(x, y) = x^2 + y^2 - 200^2$. In the hierarchical system, the follower robots are controlled to enter a ring-shaped region with $r_1 = 60$ cm and $r_2 = 20$ cm as defined in (7) and (8). Moreover, the center of the ring-shaped region is equal to the position of the leader robot.

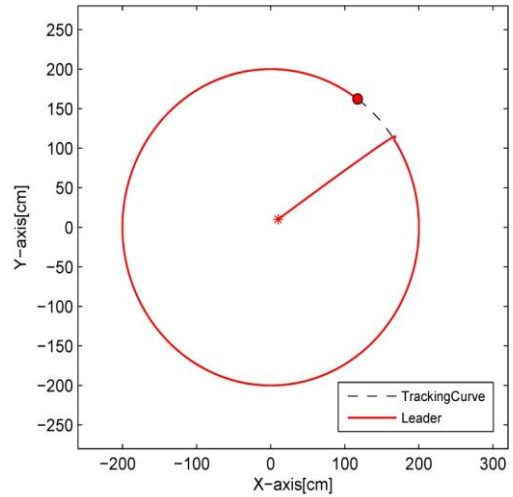


Figure 5. An illustration of the notation for the movement of the follower robots.

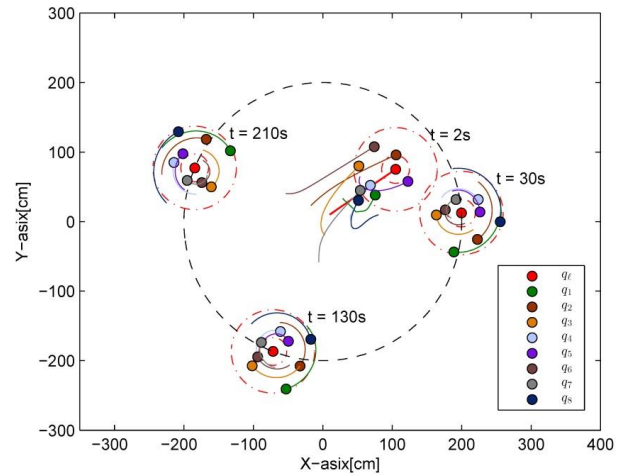


Figure 6. Snapshots of the simulation results for the proposed hierarchical multi-robot system. The black dashed circle denotes the desired curve for the leader robot.

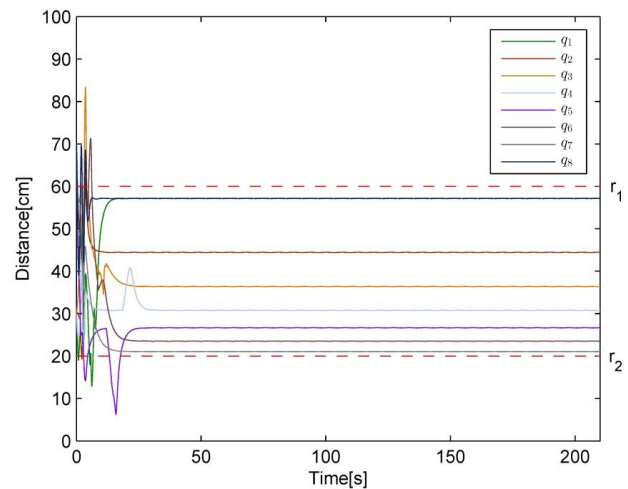


Figure 7. Distances between the leader and all follower robots. The two red dashed lines are the radii of the outer and inner circles of the designated region.

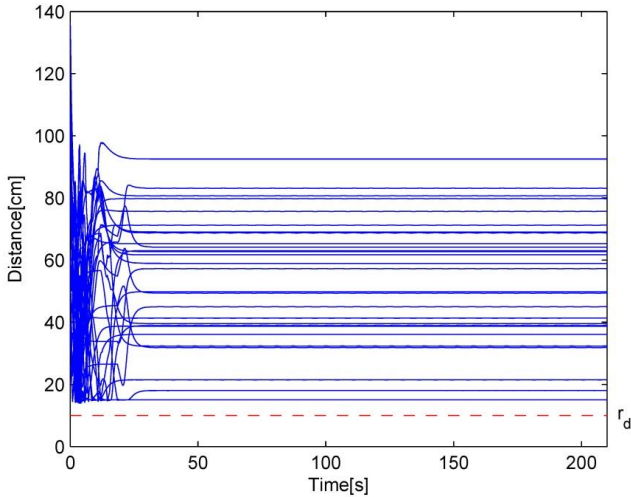


Figure 8. The distances between any pair of follower robots. The red dashed line denotes the smallest safe distance for collision avoidance.

In the simulation, the initial positions of the follower robots in the workspace are chosen randomly, while the initial position of the leader robot is located close to the origin. The control gains of the leader robot are $k_i = 5 \times 10^{-5}$, $b_\ell = 5$, and $\omega_\ell = 20$, and the control parameters for the follower robots are given as $k_1 = k_2 = 6$, $b_f = [1.75, 2.25, 2.75, 3.25, 3.75, 4.25, 4.75, 1.75]^T$, and $\omega_f = 100$. To prevent the mobile robots from approaching the periphery of the designated region when following the leader robot, the centrifugal force $\omega_f^2 \|q_{it}\| e_i^c$ is added to the control input of the follower robots. The simulation results are illustrated in Figs. 5 to 8.

Figure 5 shows that the leader robot is able to successfully approach the prescribed curve defined by $f_i(x, y)$. The red star in Fig. 5 denotes the initial position of the leader robot, and the red circle is the position of the leader robot. The leader robot circulates clockwise after converging to the curve. During the transient, there is an overshoot when the leader robot reaches the given circular curve. The percentage of overshoot is adjustable by tuning the damping coefficient b_ℓ and tracking gain k_i . The snapshots of the multi-robot system in different time slots are shown in Fig. 6. The follower robots converge to the designated region with the leader robot as the center. After entering the ring-shaped region, all the follower robots circulate clockwise within the region. The orbiting radii are different for different follower robots because, as mentioned in Remark 2, the distances to the leader robot are dependent on the damping coefficient b_{fi} . The distance between the follower robots to the leader robot is shown in Fig. 7, which demonstrates that the follower robots can stay within the designated ring-shaped region. Moreover, Fig. 8 shows that all distances between any

pair of two follower robots exceed 10cm; hence, no collision occurs in the proposed multi-robot system.

Experiments

Next, we use a group of E-puck robots [20] to execute the proposed hierarchical coordination system. The experimental framework is composed of a localization system, a personal computer (PC), and four E-puck robots. The localization system is accomplished by using an overhead camera to obtain the location of the leader and follower robots moving in the arena which measures 3.6 m \times 2.1 m. The position information is sent to a PC to calculate the control commands for the multi-robot system, and the control inputs are transmitted to the robots via Bluetooth.

In the experiments, the desired curve is a circle centered at $[x, y] = [113, 216]$ cm with a radius of 50 cm. The ring-shaped region for the follower robots is given with $r_1 = 40$ cm, $r_2 = 20$ cm. The control parameters are $k_i = 10^{-5}$, $b_\ell = 1$, $\omega_\ell = 1$, $k_1 = k_2 = 5 \times 10^{-5}$, $b_{fi} = 0.75$, $\omega_f = -1.5$ for $i = 1, 2, 3$. The sensing range of collision avoidance is $R_d = 25$ cm, the smallest safe distance is $r_d = 10$ cm, and the control gain of the collision avoidance is $k_a = 20$. The validation results are shown in Figs. 9 to 12.

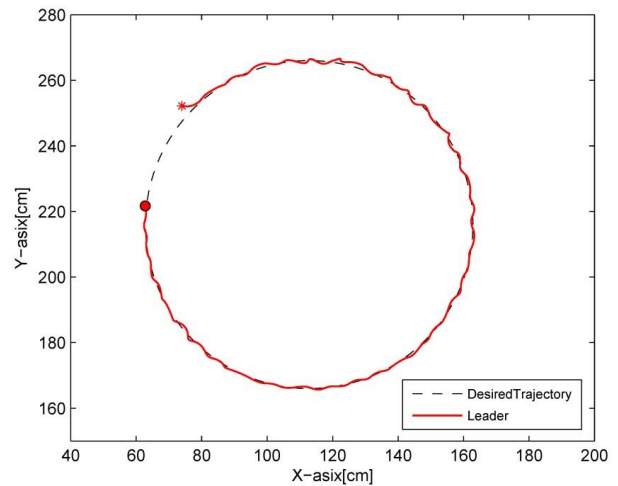


Figure 9. The trajectory of the leader robot in the experiment.

The trajectory of the leader robots is illustrated in Fig. 9, where the black dashed circle is the desired trajectory of the leader. This result demonstrates the performance of the leader robot converging to the desired curve and circulating on the curve. The snapshots of the multi-robot system are shown in Fig. 10. It can be observed that the entire control system is stable, and the follower robots are able to enter the prescribed region by moving with the leader robot. Moreover, the result shows that all the follower robots are able to circulate counterclockwise in the region while leader moves clockwise along the curve. Figures 11 and 12 illustrate the

distance from each follower robot to the leader robot and to the other follower robots. The results demonstrate that all the follower robots can stay in the given regions while avoiding collision with the other mobile robots.

Remark 3 Different from the simulation results which take the centrifugal force $\omega_f^2 \|q_{it}\| e_i^c$ into account, no

additional force is considered in the experiments for the follower robots to compensate for the centrifugal forces. Therefore, the movements of the follower robots in Figs. 11 and 12 are oscillatory and move towards the outer circle of the given region.

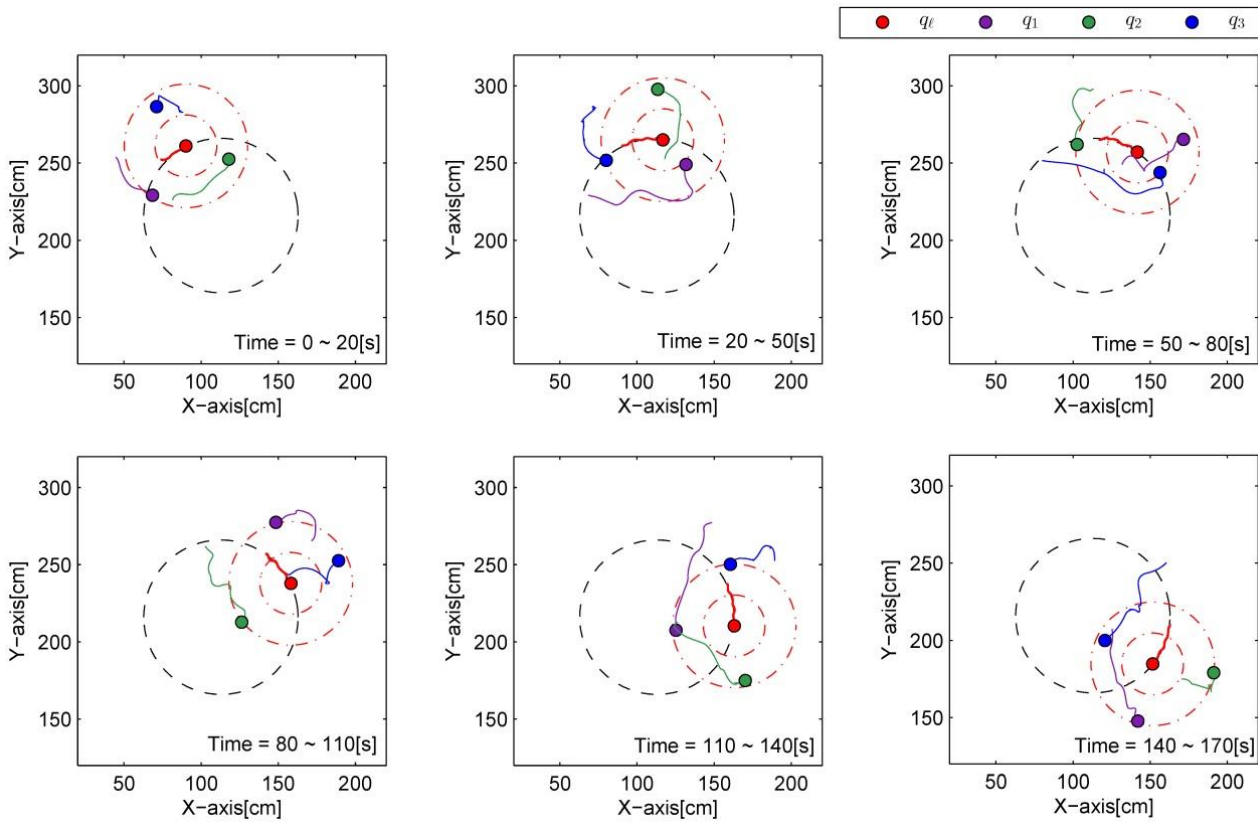


Figure 10. Snapshots of the hierarchical multi-robot system in the experiment. The red solid line denotes the trajectory of the leader robot, and the red dash-dot lines define the region for the convergence of the follower robots.

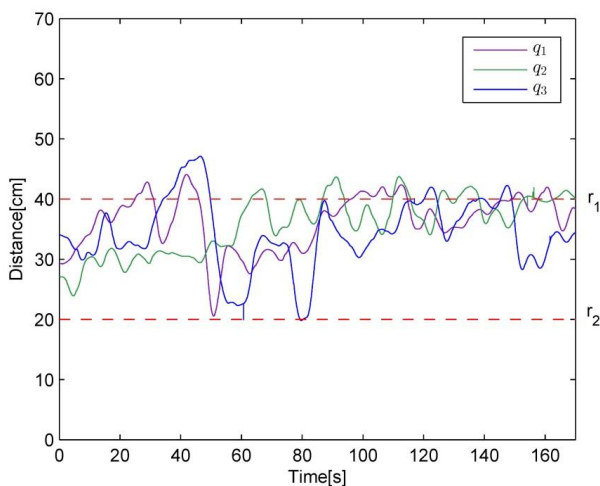


Figure 11. The distances between the leader and all the follower robots in the experiment. Two red dash lines r_1 and r_2 are radii of the outer and inner circles of the designated region.

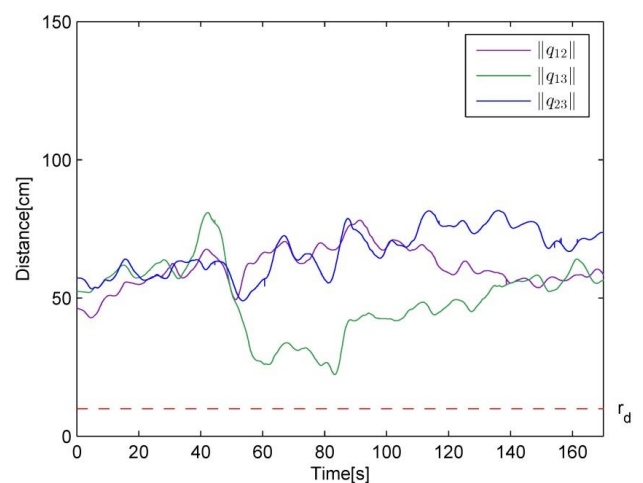


Figure 12. The distance between any pair of follower robots in the experiment. The red dashed line r_d denotes the smallest safe distance for collision avoidance.

Conclusions

A control framework for the hierarchical coordination of multi-robot systems is proposed. Based on the proposed control algorithms, the leader robot can approach a curve defined by an implicit function, and circulate along the curve. The follower robots, without receiving information about the curve, can track the leader robot by surrounding it within a given region. Using the Lyapunov method, the stability and convergence of the mobile robot network were studied. Simulation and experimental results were demonstrated the efficiency of the proposed control algorithms. Future work will further analyze the stability of the follower robots when the leader robot is orbiting, and extend the proposed system such that only a portion of the follower robots can access the leader robot information.

References

- [1] J. Alonso-Mora, A. Breitenmoser, M. Rufli, R. Siegwart, and P. Beardsley, "Multi-robot system for artistic pattern formation," in proceeding of *IEEE International Conference on Robotics and Automation*, Shanghai, China, May 9-13, 2011, pp. 4512–4517.
doi: [10.1109/ICRA.2011.5980269](https://doi.org/10.1109/ICRA.2011.5980269)
- [2] T. Balch and R. C. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp.926–939, 1998.
doi: [10.1109/70.736776](https://doi.org/10.1109/70.736776)
- [3] R. W. Beard, J. Lawton, and F. Y. Hadaegh, "A feedback architecture for formation control," in proceeding of *American Control Conference*, Chicago, IL, USA, June 28-30, 2000, pp. 4087– 4091.
doi: [10.1109/ACC.2000.876990](https://doi.org/10.1109/ACC.2000.876990)
- [4] R. Brooks, "A robust layered control system for a mobile robot," *IEEE Journal on Robotics and Automation*, vol. 2, no. 1, pp. 14–23, 1986.
doi: [10.1109/JRA.1986.1087032](https://doi.org/10.1109/JRA.1986.1087032)
- [5] C.-C. Cheah, S.-P. Hou, and J.-J. E. Slotine, "Region-based shape control for a swarm of robots," *Automatica*, vol. 45, no. 10, pp. 2406–2411, 2009.
doi: [10.1016/j.automatica.2009.06.026](https://doi.org/10.1016/j.automatica.2009.06.026)
- [6] A. K. Das, R. Fierro, V. Kumar, J. P. Ostrowski, J. Spletzer, and C. J. Taylor, "A vision-based formation control framework," *IEEE Transactions on Robotics and Automation*, vol. 18, no.5, pp.813–825, Oct. 2002.
doi: [10.1109/TRA.2002.803463](https://doi.org/10.1109/TRA.2002.803463)
- [7] M. Defoort, T. Floquet, A. Kokosy, and W. Perruquetti, "Slidingmode formation control for cooperative autonomous mobile robots," *IEEE Transactions on Industrial Electronics*, vol. 55, no. 11, pp. 3944–3953, Nov. 2008.
doi: [10.1109/TIE.2008.2002717](https://doi.org/10.1109/TIE.2008.2002717)
- [8] W. Dong, "Flocking of multiple mobile robots based on backstepping," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 41, no. 2, pp. 414–424, 2011.
doi: [10.1109/TSMCB.2010.2056917](https://doi.org/10.1109/TSMCB.2010.2056917)
- [9] W. Dong and J. A. Farrell, "Cooperative control of multiple nonholonomic mobile agents," *IEEE Transactions on Automatic Control*, vol. 53, no. 6, pp.1434–1448, 2008.
doi: [10.1109/TAC.2008.925852](https://doi.org/10.1109/TAC.2008.925852)
- [10] J. T. Feddema, C. Lewis, and D. A. Schoenwald, "Decentralized control of cooperative robotic vehicles: theory and application," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 852–864, 2002.
doi: [10.1109/TRA.2002.803466](https://doi.org/10.1109/TRA.2002.803466)
- [11] V. M. Goncalves, L. C. A. Pimenta, C. A. Maia, B. C. O. Dutra, and G. A. S. Pereira, "Vector fields for robot navigation along timevarying curves in n-dimensions," *IEEE Transactions on Robotics*, vol. 26, no. 4, pp. 647–659, 2010.
doi: [10.1109/TRO.2010.2053077](https://doi.org/10.1109/TRO.2010.2053077)
- [12] R. Haghghi and C.-C. Cheah, "Multi-group coordination control for robot swarms," *Automatica*, vol. 48, no. 10, pp. 2526–2534, Oct. 2012.
doi: [10.1016/j.automatica.2012.03.028](https://doi.org/10.1016/j.automatica.2012.03.028)
- [13] S.-P. Hou and C.-C. Cheah, "Dynamic compound shape control of robot swarm," *IET Control Theory & Applications*, vol. 6, no. 3, pp. 454–460, 2012.
doi: [10.1049/iet-cta.2011.0115](https://doi.org/10.1049/iet-cta.2011.0115)
- [14] M.-A. Hsieh, V. Kumar, and L. Chaimowicz, "Decentralized controllers for shape generation with robotic swarms," *Robotica*, vol. 26, no. 5, pp. 691–701, 2008.
doi: [10.1017/S0263574708004323](https://doi.org/10.1017/S0263574708004323)
- [15] M.-A. Hsieh, S. Loizou, and V. Kumar, "Stabilization of multiple robots on stable orbits via local sensing," In *IEEE International Conference on Robotics and Automation*, pp. 2312–2317, 2007.
doi: [10.1109/ROBOT.2007.363664](https://doi.org/10.1109/ROBOT.2007.363664)
- [16] M. A. Lewis and K.-H. Tan, "High precision formation control of mobile robots using virtual structures," *Autonomous Robots*, vol. 4, no. 4, pp. 387–403, Oct. 1997.
doi: [10.1023/A:1008814708459](https://doi.org/10.1023/A:1008814708459)
- [17] X.-H. Li, J.-Z. Xiao, and Z.-J. Cai, "Backstepping based multiple mobile robots formation control," in proceeding of *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Edmonton, Alta., Canada, Aug. 2-6, 2005, pp. 887–892.
doi: [10.1109/IROS.2005.1545161](https://doi.org/10.1109/IROS.2005.1545161)
- [18] Y.-C. Liu, "Task-space control of bilateral human-swarm interaction with constant time delays," in proceeding of *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Tokyo, Japan, Nov. 3-7, 2013, pp. 1663–1669.
doi: [10.1109/IROS.2013.6696572](https://doi.org/10.1109/IROS.2013.6696572)



- [19] Y.-C. Liu and N. Chopra, "Control of semi-autonomous teleoperation system with time delays," *Automatica*, vol. 49, no. 6, pp. 1553–1565, Jun. 2013. doi: [10.1016/j.automatica.2013.02.009](https://doi.org/10.1016/j.automatica.2013.02.009)
- [20] F. Mondada, M. Bonani, X. Raemy, J. Pugh, C. Cianci, A. Klaptocz, S. Magnenat, J.-C. Zufferey, D. Floreano, and A. Martinoli, "The e-puck, a robot designed for education in engineering," in proceeding of *9th Conference on Autonomous Robot Systems and Competitions*, Castelo Branco, Portugal, May 7, 2009, pp. 59–65. <http://infoscience.epfl.ch/record/135236>
- [21] L. C. A. Pimenta, G. A. S. Pereira, M. M. Gonçalves, N. Michael, M. Turpin, and V. Kumar, "Decentralized controllers for perimeter surveillance with teams of aerial robots," *Advanced Robotics*, vol. 27, no. 9, pp. 697–709, 2013. doi: [10.1080/01691864.2013.778942](https://doi.org/10.1080/01691864.2013.778942)
- [22] E. Rimon and D. E. Koditschek, "Exact robot navigation using artificial potential functions," *IEEE Transactions on Robotics and Automation*, vol. 8, no. 5, pp. 201–212, 1992. doi: [10.1109/70.163777](https://doi.org/10.1109/70.163777)
- [23] L. Sabattini, C. Secchi, and C. Fantuzzi, "Closed-curve path tracking for decentralized systems of multiple mobile robots," *Journal of Intelligent & Robotic Systems*, vol. 71, no. 1, pp. 109–123, 2013. doi: [10.1007/s10846-012-9763-9](https://doi.org/10.1007/s10846-012-9763-9)
- [24] L. A. Sanchez-Porras and Y.-C. Liu, "Region-based curve tracking control for leader-follower mobile robot networks," in proceeding of *International Conference on Advanced Robotics and Intelligent Systems*, Taipei, Taiwan, June 6-8, 2014, pp. 22–27. doi: [10.1109/ARIS.2014.6871495](https://doi.org/10.1109/ARIS.2014.6871495)
- [25] B. Siciliano and O. Khatib, Springer handbook of robotics, Springer, 2008.
- [26] J.-J. E. Slotine and W. Li., Applied nonlinear control, 1991.
- [27] K. Sugihara and I. Suzuki, Distributed algorithms for controlling multiple mobile robots, Technical report, DTIC Document, 1994.
- [28] P. Svestka and M. H. Overmars, "Coordinated path planning for multiple robots," *Robotics and Autonomous Systems*, vol. 23, no. 3, pp. 125–152, Apr. 1998. doi: [10.1016/S0921-8890\(97\)00033-X](https://doi.org/10.1016/S0921-8890(97)00033-X)
- [29] E. K. Xidias and P. N. Azariadis, "Mission design for a group of autonomous guided vehicles," *Robotics and Autonomous Systems*, vol. 59, no. 1, pp. 34–43, Jan. 2011. doi: [10.1016/j.robot.2010.10.003](https://doi.org/10.1016/j.robot.2010.10.003)
- [30] H. Yamaguchi, "A distributed motion coordination strategy for multiple nonholonomic mobile robots in cooperative hunting operations," *Robotics and Autonomous Systems*, vol. 43, no. 4, pp. 257–282, 2003. doi: [10.1109/CDC.2002.1184310](https://doi.org/10.1109/CDC.2002.1184310)
- [31] B. J. Young, R. W. Beard, and J. M. Kelsey, "A control scheme for improving multi-vehicle formation maneuvers," in proceeding of *American Control Conference*, Arlington, VA, USA, June 25-27, 2001, pp. 704–709. doi: [10.1109/ACC.2001.945797](https://doi.org/10.1109/ACC.2001.945797)

